Necessary and sufficient conditions for some two variable orthogonal designs in order 36

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Dedicated to Ralph Gordon Stanton on his 70th birthday

Abstract

We give new sets of sequences with entries from $\{0, \pm a, \pm b, \pm c, \pm d\}$ on the commuting variables a, b, c, d with zero autocorrelation function. We show that the necessary conditions are sufficient for the existence two variable orthogonal designs constructed from circulant matrices in order 36.

Further we show that the necessary conditions for the existence of an $OD(36; s_1, s_2)$ are sufficient except possibly for the following five cases:

$$(3,29) \quad (11,20) \quad (11,21) \quad (13,19) \quad (15,17).$$

Key words and phrases: Autocorrelation, construction, sequence, orthogonal design.

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1 Introduction

Throughout this paper we will use the definition and notation of Koukouvinos, Mitrouli, Seberry and Karabelas [2].

We note the following lemma which has not previously been published but which is useful in determining the size of programs to search for orthogonal designs. The result is obtained by simple counting.

Lemma 1 *Let n be the order of an orthogonal design then the number of cases which must be studied to determine whether all two variable orthogonal designs exist is $\frac{1}{4}n^2$.*

2 New orthogonal designs

Theorem 1 *There are $OD(36; s_1, s_2, s_3, s_4)$ constructed using four circulant matrices in the Goethals-Seidel array for the following (new) 4-tuples*

(1, 1, 1, 25)	(1, 1, 2, 32)	(1, 1, 4, 25)	(1, 1, 5, 20)	(1, 1, 9, 25)
(1, 1, 16, 16)	(1, 1, 17, 17)	(1, 2, 2, 25)	(1, 2, 3, 24)	(1, 2, 6, 27)
(1, 3, 8, 24)	(2, 2, 4, 25)	(2, 2, 8, 18)	(2, 2, 16, 16)	(2, 4, 4, 18)
(2, 4, 8, 16)	(2, 8, 10, 10)	(4, 4, 8, 18)	(5, 5, 10, 10)	(8, 8, 8, 8)
(9, 9, 9, 9)				

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Theorem 2 *An $OD(36; s_1, s_2)$ cannot exist for the following 2−tuples (s_1, s_2);*

(1,7)	(1,15)	(1,23)	(1,28)	(1,31)	(2,14)	(2,30)
(3,5)	(3,13)	(3,20)	(3,21)	(4,7)	(4,15)	(4,23)
(4,28)	(4,31)	(5,11)	(5,12)	(5,19)	(5,27)	(6,10)
(6,26)	(7,9)	(7,16)	(7,17)	(7,25)	(7,28)	(8,14)
(9,15)	(9,23)	(10,17)	(10,22)	(10,24)	(11,13)	(11,16)
(12,13)	(12,15)	(12,20)	(12,21)	(12,23)	(14,18)	(15,16)
(15,20)	(16,19)					

An $OD(4N; 15, 20)$ may exist in orders $4N$, $N > 9$.

Proof. These cases are eliminated by the number theoretic necessary conditions given in [1] or [2, Lemma 3].

Theorem 3 *There are no orthogonal designs $OD(4n; s_1, s_2)$ where (s_1, s_2) is one of the 2-tuples*

(3,29) (11,20) (11,21) (13,19) (15,17)

constructed using four circulant matrices in the Goethals-Seidel array.

Proof. There is no an integer sum-fill matrix P as described in [2, Lemma 3]. □

Theorem 4 *There are $4 - NPAF(s_1, s_2)$ of length 9 for the following 2-tuples where $29 \leq s_1 + s_2 \leq 36$:*

(1,26)	(1,32)	(1,33)	(2,25)	(2,28)	(2,31)	(2,32)
(2,34)	(3,26)	(3,28)	(3,30)	(4,25)	(4,26)	(4,27)
(4,29)	(4,30)	(4,32)	(5,24)	(5,25)	(5,26)	(5,28)
(5,29)	(5,31)	(6,23)	(6,24)	(7,22)	(7,23)	(7,24)
(7,26)	(8,19)	(8,21)	(8,22)	(8,23)	(8,24)	(8,25)
(8,26)	(9,20)	(9,21)	(9,22)	(9,24)	(9,27)	(10,20)
(10,21)	(10,23)	(10,25)	(11,18)	(11,19)	(11,22)	(11,23)
(11,24)	(12,17)	(12,18)	(12,19)	(12,22)	(12,24)	(13,14)
(13,17)	(13,18)	(13,20)	(13,21)	(13,22)	(13,23)	(14,16)
(14,19)	(14,20)	(14,21)	(14,22)	(15,15)	(15,18)	(15,19)
(15,21)	(16,16)	(16,17)	(16,18)	(16,20)	(17,17)	(18,18)

Lemma 2 *An exhaustive search shows that $4 - NPAF(s_1, s_2)$ of length 9 do not exist for the 2−tuples (3,31) and (5,30).*

Theorem 5 *The sequences given in the Appendices can be used to construct the appropriate designs to establish that the necessary conditions for the existence of an $OD(36; s_1, s_2)$ are sufficient. except possibly for the following five cases:*

(3,29) (11,20) (11,21) (13,19) (15,17).

4−tuple	weight	ref	n	4−tuple	weight	ref	n	4−tuple	weight	ref	n
1 1 1 1	4	[GS]	1	1 4 8 18	31			2 8 8 18	36		
1 1 1 4	7	[GS]	3	1 4 9 9	23			2 8 9 9	28		
1 1 1 9	12	[GS]	5	1 4 9 16	30			2 8 10 10	30	AppB	9
1 1 1 16	19	[GS]	*	1 4 10 10	25		10	2 8 13 13	36		
1 1 1 25	28	AppA	P	1 4 13 13	31			3 3 3 3	12	[GS]	3
1 1 2 2	6	[GS]	2	1 5 5 9	20	[2]	5	3 3 3 12	21	[2]	6
1 1 2 8	12	[GS]	3	1 5 5 16	27			3 3 3 27	36		12
1 1 2 18	22	[2]	6	1 5 5 25	36			3 3 6 6	18	[GS]	5
1 1 2 32	36	AppA	P	1 6 8 12	27			3 3 6 24	36		12
1 1 4 4	10	[GS]	3	1 8 8 9	26			3 3 12 12	30		12
1 1 4 9	15	[2]	5	1 8 8 16	33			3 3 15 15	36		12
1 1 4 16	22	[GS]	7	1 8 9 18	36			3 4 6 8	21		
1 1 4 25	31	AppA	P	1 9 9 9	28			3 4 6 18	31		
1 1 5 5	12	[GS]	3	1 9 10 10	30			3 6 6 12	27		12
1 1 5 20	27	AppA	P	1 9 13 13	36			3 6 8 9	26		
1 1 8 8	18	[GS]	5	2 2 2 2	8	[GS]	2	3 6 8 16	33		
1 1 8 18	28			2 2 2 8	14	[GS]	5	3 6 9 18	36		12
1 1 9 9	20	[GS]	5	2 2 2 18	24	[2]	7	3 8 10 15	36		
1 1 9 16	27			2 2 4 4	12	[GS]	5	4 4 4 4	16	[GS]	4
1 1 9 25	36	AppA	P	2 2 4 9	17	[2]	5	4 4 4 9	21		
1 1 10 10	22	[2]	6	2 2 4 16	24	[GS]	6	4 4 4 16	28	[GS]	7
1 1 13 13	28	[GS]	*	2 2 4 25	33	AppA	P	4 4 5 5	18	[GS]	5
1 1 16 16	34	AppB	9	2 2 5 5	14	[GS]	6	4 4 5 20	33		
1 1 17 17	36	AppA	P	2 2 5 20	29			4 4 8 8	24	[GS]	6
1 2 2 4	9	[GS]	3	2 2 8 8	20	[GS]	5	4 4 8 18	34	AppB	9
1 2 2 9	14	[GS]	5	2 2 8 18	30	AppB	9	4 4 9 9	26		
1 2 2 16	21	[2]	7	2 2 9 9	22	[GS]	6	4 4 9 16	33		
1 2 2 25	30	AppA	P	2 2 9 16	29			4 4 10 10	28	[GS]	7
1 2 3 6	12	[GS]	3	2 2 10 10	24	[GS]	6	4 4 13 13	34		
1 2 3 24	30	AppA	P	2 2 13 13	30		14	4 5 5 9	23		
1 2 4 8	15	[GS]	5	2 2 16 16	36	AppB	9	4 5 5 16	30		
1 2 4 18	25	[2]	7	2 3 4 6	15	[GS]	5	4 6 8 12	30		10
1 2 6 12	21	[GS]	7	2 3 4 24	33			4 8 8 9	29		
1 2 6 27	36	AppA	P	2 3 6 9	20	[2]	5	4 8 8 16	36		10
1 2 8 9	20	[2]	5	2 3 6 16	27			4 9 9 9	31		
1 2 8 16	27			2 3 6 25	36			4 9 10 10	33		
1 2 8 25	36			2 3 10 15	30			5 5 5 5	20	[GS]	5
1 2 9 18	30			2 4 4 8	18	[GS]	5	5 5 8 8	26	[GS]	7
1 2 11 22	36			2 4 4 18	28	AppB	9	5 5 8 18	36		
1 3 6 8	18	[2]	6	2 4 6 12	24	[GS]	6	5 5 9 9	28		
1 3 6 18	28	[2]	7	2 4 8 9	23	[2]	7	5 5 10 10	30	AppB	9
1 3 8 24	36	AppA	P	2 4 8 16	30	AppB	9	5 5 13 13	36		14
1 4 4 4	13	[GS]	5	2 4 9 18	33			6 6 6 6	24	[GS]	6
1 4 4 9	18			2 5 5 8	20	[GS]	5	6 6 12 12	36		10
1 4 4 16	25	[2]	7	2 5 5 18	30			7 7 7 7	28	[GS]	7
1 4 4 25	34			2 6 7 21	36			8 8 8 8	32	[GS]	8
1 4 5 5	15	[GS]	5	2 6 9 12	29			8 8 9 9	34		
1 4 5 20	30			2 6 12 16	36		10	8 8 10 10	36		10
1 4 8 8	21	[GS]	6	2 8 8 8	26	[GS]	7	9 9 9 9	36	AppB	9

Table 1: The existence of $OD(36; s_1, s_2, s_3, s_4)$. There are 351 possible 4−tuples. 154 satisfy the number theoretic necessary conditions for the existence of 4−variable designs: 197 do not. Designs are known to exist in 79 cases: 53 cases are unresolved.

P indicates that there are 4-PAF sequences of length 9.

* indicates that there are 4-PAF sequences for every odd length ≥ 7.

n indicates that there are $4 - NPAF$ sequences giving the design for every length $\geq n$.

s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n	s_1	s_2	n
1	1	1	1	2	1	1	3	1	1	4	2	1	5	2	1	6	3	1	7	NE
1	8	3	1	9	3	1	10	3	1	11	3	1	12	4	1	13	5	1	14	5
1	15	NE	1	16	5	1	17	5	1	18	5	1	19	5	1	20	7	1	21	7
1	22	7	1	23	NE	1	24	7	1	25	7	1	26	9	1	27	7	1	28	NE
1	29	P	1	30	P	1	31	NE	1	32	9	1	33	9	1	34	P	1	35	P
2	2	1	2	3	2	2	4	2	2	5	3	2	6	2	2	7	3	2	8	3
2	9	5	2	10	3	2	11	5	2	12	5	2	13	5	2	14	NE	2	15	5
2	16	5	2	17	5	2	18	5	2	19	7	2	20	6	2	21	7	2	22	7
2	23	7	2	24	7	2	25	9	2	26	7	2	27	P	2	28	9	2	29	P
2	30	NE	2	31	9	2	32	9	2	33	P	2	34	9	3	3	2	3	4	3
3	5	NE	3	6	3	3	7	3	3	8	3	3	9	3	3	10	5	3	11	5
3	12	5	3	13	NE	3	14	5	3	15	5	3	16	7	3	17	5	3	18	7
3	19	7	3	20	NE	3	21	NE	3	22	7	3	23	7	3	24	7	3	25	7
3	26	9	3	27	P	3	28	9	3	29	Y	3	30	9	3	31	P	3	32	P
3	33	P	4	4	2	4	5	3	4	6	3	4	7	NE	4	8	3	4	9	5
4	10	5	4	11	5	4	12	5	4	13	5	4	14	5	4	15	NE	4	16	5
4	17	7	4	18	7	4	19	7	4	20	7	4	21	7	4	22	7	4	23	NE
4	24	7	4	25	9	4	26	9	4	27	9	4	28	NE	4	29	9	4	30	9
4	31	NE	4	32	9	5	5	3	5	6	3	5	7	3	5	8	5	5	9	5
5	10	5	5	11	NE	5	12	NE	5	13	5	5	14	5	5	15	5	5	16	7
5	17	7	5	18	7	5	19	NE	5	20	7	5	21	7	5	22	P	5	23	7
5	24	9	5	25	9	5	26	9	5	27	NE	5	28	9	5	29	9	5	30	P
5	31	9	6	6	3	6	7	5	6	8	5	6	9	5	6	10	NE	6	11	5
6	12	5	6	13	7	6	14	5	6	15	7	6	16	7	6	17	7	6	18	7
6	19	7	6	20	7	6	21	7	6	22	7	6	23	9	6	24	9	6	25	9
6	26	NE	6	27	P	6	28	P	6	29	P	6	30	P	7	7	4	7	8	5
7	9	NE	7	10	5	7	11	7	7	12	7	7	13	5	7	14	7	7	15	7
7	16	NE	7	17	NE	7	18	7	7	19	9	7	20	P	7	21	7	7	22	9
7	23	9	7	24	9	7	25	NE	7	26	9	7	27	P	7	28	NE	7	29	P
8	8	5	8	9	5	8	10	5	8	11	5	8	12	5	8	13	7	8	14	NE
8	15	7	8	16	7	8	17	7	8	18	7	8	19	9	8	20	7	8	21	9
8	22	9	8	23	9	8	24	9	8	25	9	8	26	9	8	27	P	8	28	P
9	9	5	9	10	5	9	11	5	9	12	7	9	13	6	9	14	7	9	15	NE
9	16	7	9	17	7	9	18	7	9	19	7	9	20	9	9	21	9	9	22	9
9	23	NE	9	24	9	9	25	P	9	26	P	9	27	9	10	10	5	10	11	7
10	12	7	10	13	7	10	14	7	10	15	7	10	16	7	10	17	NE	10	18	7
10	19	P	10	20	9	10	21	9	10	22	NE	10	23	9	10	24	NE	10	25	9
10	26	P	11	11	6	11	12	7	11	13	NE	11	14	7	11	15	7	11	16	NE
11	17	7	11	18	9	11	19	9	11	20	Y	11	21	Y	11	22	9	11	23	9
11	24	9	11	25	P	12	12	7	12	13	NE	12	14	7	12	15	NE	12	16	7
12	17	9	12	18	9	12	19	9	12	20	NE	12	21	NE	12	22	9	12	23	NE
12	24	9	13	13	7	13	14	9	13	15	7	13	16	P	13	17	9	13	18	9
13	19	Y	13	20	9	13	21	9	13	22	P	13	23	9	14	14	7	14	15	P
14	16	9	14	17	P	14	18	NE	14	19	9	14	20	9	14	21	9	14	22	9
15	15	9	15	16	NE	15	17	Y	15	18	9	15	19	9	15	20	NE	15	21	9
16	16	9	16	17	9	16	18	9	16	19	NE	16	20	9	17	17	9	17	18	P
17	19	P	18	18	9															

Table 2: The existence of $OD(36; s_1, s_2)$. There are 324 possible 2–tuples, 275 correspond to designs which exist: 44 correspond to designs eliminated by number theory (NE). For 5 cases, if the designs exist, they cannot be constructed using circulant matrices (Y).
P indicates that $4 - PAF$ sequences with length 9 exist; n indicates $4 - NPAF$ sequences with length n exist.

Lemma 3 *The sequences given in the Appendices can be used to construct the appropriate designs to establish that the necessary conditions for the existence of an $OD(36; s_1, s_2)$ constructed using four circulant matrices in the Goethals-Seidel construction are sufficient.*

References

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Appendix A: Order 36 (Sequences with zero periodic autocorrelation function)

Design	A_1									A_2, A_3									A_4								
(1, 1, 1, 25)	−d	d	d	d	−d	−d	−d	d	a	−d	d	0	0	−d	d	−d	b	d									
										−d	0	−d	c	d	0	d	0	0	0	d	d	−d	0	d	d	d	d
(1, 1, 2, 32)	−d	d	d	d	−d	−d	−d	d	a	−d	d	−d	d	−d	−d	b	d	d									
										−d	−d	−d	−d	−d	d	−d	c	d	−d	d	d	d	d	d	−d	c	d
(1, 1, 4, 25)	−d	d	0	0	−d	d	−d	a	d	−d	0	0	d	−d	−d	b	d	d									
										c	0	c	d	d	−d	d	−d	−d	d	d	d	d	d	d	c	−d	−c
(1, 1, 5, 20)	d	c	−c	−d	0	0	a	0	0	−d	d	d	0	0	b	0	0	−d									
										−d	d	−d	−d	d	−d	−d	−d	c	0	−d	d	−d	c	c	d	d	d
(1, 1, 9, 25)	−c	a	c	d	d	−d	d	−d	−d	−d	d	d	d	−d	−d	−d	d	b									
										c	d	−d	d	−d	c	c	c	−c	c	d	d	d	d	−c	d	d	−d
(1, 1, 17, 17)	a	−c	−c	d	−d	d	−d	c	c	b	−c	d	c	−c	c	−c	−d	c									
										c	c	−d	c	d	c	−d	d	d	−c	−d	c	d	c	−d	−d	−d	−d
(1, 2, 2, 25)	a	0	−d	d	−d	d	−d	d	0	b	−d	−d	d	c	−d	d	d	0									
										b	d	d	−d	−c	d	−d	−d	0	−d	0	d	d	d	d	d	d	0
(1, 2, 3, 24)	a	−d	d	d	d	−d	−d	−d	d	b	−c	0	d	d	−d	d	0	0									
										b	c	0	−d	d	−d	−d	0	0	c	−d	d	−d	d	d	d	d	d
(1, 2, 6, 27)	a	−c	−d	−d	−d	d	d	d	c	b	c	d	−d	d	d	d	−d	d									
										b	−c	−d	d	−d	−d	−d	d	−d	d	−d	−d	c	−d	c	−d	−d	d
(1, 3, 8, 24)	a	−d	−d	−d	d	−d	d	d	d	b	−d	d	d	c	−d	d	c	d									
										b	−d	d	−d	d	d	−c	−c	d	−c	b	c	−d	c	−c	−d	−d	−d
(2, 2, 4, 25)	a	b	−d	−d	d	−d	d	d	0	a	−b	−d	−d	d	−d	d	d	0									
										d	−c	−d	−d	d	d	−c	−d	0	d	d	d	d	c	d	−d	d	−c

Appendix A(cont): Order 36 (Sequences with zero periodic autocorrelation function)

Design	A_1									A_2, A_3									A_4								
(3, 31)	a	$-b$	$-b$	$-b$	b	b	b	b	b	b	a	$-b$	b	$-b$	b	$-b$	$-b$	0									
										$-b$	a	b	$-b$	b	$-b$	$-b$	b	0	$-b$	b	b	b	$-b$	b	b	b	b
(5, 30)	a	a	b	$-b$	b	b	$-b$	$-b$	$-b$	a	$-a$	$-b$	$-b$	$-b$	$-a$	b	b	$-b$									
										b	$-b$	$-b$	b	$-b$	$-b$	$-b$	$-b$	$-b$	b	b	$-b$	b	$-b$	b	$-b$	$-b$	0
(9, 20)	a	a	$-b$	a	b	$-b$	$-b$	b	b	b	$-a$	b	$-b$	$-b$	b	a	b	0									
										b	$-b$	b	$-b$	$-b$	$-b$	$-b$	$-b$	0	a	$-a$	0	a	0	0	$-a$	0	0
(9, 22)	a	a	$-b$	a	b	$-b$	$-b$	b	b	b	$-b$	b	$-b$	$-b$	b	$-b$	$-b$	$-b$									
										b	a	$-a$	$-b$	$-b$	$-b$	$-b$	0	0	a	$-a$	$-a$	0	$-b$	a	$-b$	0	0
(10, 19)	a	b	0	b	0	0	b	0	0	a	b	a	a	$-b$	b	$-b$	$-b$	0									
										b	b	$-a$	a	$-b$	b	$-b$	$-b$	0	$-a$	b	b	$-b$	$-a$	a	a	b	b
(11, 18)	a	$-b$	b	b	$-b$	b	b	b	b	$-b$	$-a$	a	$-a$	$-a$	a	a	a	0									
										$-b$	0	0	a	0	0	a	0	a	$-b$	$-b$	b	b	b	$-b$	b	$-b$	0
(11, 19)	a	b	b	b	$-b$	b	0	b	0	$-b$	$-a$	a	$-a$	$-a$	a	a	a	0									
										a	$-b$	a	$-b$	b	a	0	0	0	$-b$	$-b$	b	$-b$	$-b$	b	b	b	b
(12, 17)	a	a	0	b	0	0	b	0	0	b	$-b$	a	a	$-b$	$-b$	$-b$	b	0									
										b	$-a$	$-b$	$-b$	b	$-a$	$-b$	b	0	a	a	$-a$	a	$-a$	$-b$	$-b$	$-a$	$-b$
(13, 16)	a	$-b$	$-b$	a	b	b	$-b$	b	0	a	a	b	a	$-b$	$-b$	b	$-b$	b									
										$-a$	b	b	0	b	b	0	a	0	$-a$	$-a$	0	a	$-a$	a	0	a	0
(13, 17)	a	a	$-b$	b	b	b	b	$-b$	b	a	0	0	a	$-b$	0	$-b$	0	0									
										a	$-b$	$-b$	b	a	$-b$	b	b	0	a	b	a	$-a$	$-a$	$-a$	a	$-a$	b
(13, 22)	a	a	b	$-b$	$-b$	b	b	b	b	$-a$	a	$-b$	$-b$	b	a	a	a	$-b$									
										$-a$	b	$-a$	a	a	$-a$	a	b	b	$-b$	$-b$	b	$-b$	b	$-b$	b	b	0
(14, 15)	a	$-b$	b	$-b$	$-b$	b	b	b	0	a	a	0	$-b$	0	0	$-b$	0	0									
										a	b	$-a$	a	$-b$	a	a	b	0	$-a$	$-a$	b	a	$-a$	a	b	a	b
(14, 17)	a	a	a	$-b$	b	$-b$	0	$-b$	0	a	b	b	a	b	$-b$	0	b	0									
										a	$-a$	$-b$	$-a$	b	b	b	$-b$	$-b$	a	$-b$	a	$-a$	$-b$	a	$-a$	$-a$	0

Appendix B: Order 36 (Sequences with zero non-periodic autocorrelation function)

Design	A_1	$A_2,\ A_3$	A_4
(1, 1, 16, 16)	$a\ \ a\ \ b\ \ -b\ \ c\ \ b\ \ -b\ \ -a\ \ -a$	$a\ \ a\ \ b\ \ -b\ \ 0\ \ -b\ \ b\ \ a\ \ a$	
		$a\ \ -a\ \ b\ \ b\ \ d\ \ -b\ \ -b\ \ a\ \ -a$	$a\ \ -a\ \ b\ \ b\ \ 0\ \ b\ \ b\ \ -a\ \ a$
(2, 2, 8, 18)	$d\ \ a\ \ -d\ \ d\ \ c\ \ 0\ \ c\ \ -d\ \ 0$	$d\ \ a\ \ -d\ \ -d\ \ -c\ \ 0\ \ -c\ \ d\ \ 0$	
		$d\ \ b\ \ -d\ \ d\ \ c\ \ d\ \ -c\ \ d\ \ 0$	$d\ \ b\ \ -d\ \ -d\ \ -c\ \ -d\ \ c\ \ d\ \ 0$
(2, 2, 16, 16)	$a\ \ c\ \ c\ \ c\ \ -c\ \ d\ \ d\ \ -d\ \ d$	$a\ \ -c\ \ -c\ \ -c\ \ c\ \ -d\ \ -d\ \ d\ \ -d$	
		$b\ \ d\ \ d\ \ d\ \ -d\ \ -c\ \ -c\ \ c\ \ -c$	$b\ \ -d\ \ -d\ \ -d\ \ d\ \ c\ \ c\ \ -c\ \ c$
(2, 4, 4, 18)	$d\ \ 0\ \ a\ \ 0\ \ -d\ \ b\ \ d\ \ c\ \ -d$	$d\ \ 0\ \ a\ \ 0\ \ -d\ \ -b\ \ -d\ \ -c\ \ d$	
		$d\ \ 0\ \ d\ \ 0\ \ d\ \ b\ \ -d\ \ -c\ \ d$	$d\ \ 0\ \ d\ \ 0\ \ d\ \ -b\ \ d\ \ c\ \ -d$
(2, 4, 8, 16)	$d\ \ a\ \ -d\ \ d\ \ c\ \ d\ \ c\ \ 0\ \ 0$	$d\ \ a\ \ -d\ \ d\ \ -c\ \ d\ \ -c\ \ 0\ \ 0$	
		$d\ \ a\ \ -d\ \ -d\ \ 0\ \ -d\ \ c\ \ b\ \ -c$	$d\ \ a\ \ -d\ \ -d\ \ 0\ \ -d\ \ -c\ \ -b\ \ c$
(2, 8, 10, 10)	$c\ \ c\ \ -c\ \ d\ \ b\ \ d\ \ b\ \ 0\ \ 0$	$c\ \ c\ \ -c\ \ d\ \ -b\ \ d\ \ -b\ \ 0\ \ 0$	
		$d\ \ d\ \ -d\ \ -c\ \ 0\ \ -c\ \ b\ \ a\ \ -b$	$d\ \ d\ \ -d\ \ -c\ \ 0\ \ -c\ \ -b\ \ -a\ \ b$
(4, 4, 8, 18)	$d\ \ c\ \ 0\ \ c\ \ -d\ \ d\ \ -b\ \ -d\ \ a$	$d\ \ c\ \ 0\ \ c\ \ -d\ \ -d\ \ b\ \ d\ \ -a$	
		$d\ \ c\ \ d\ \ -c\ \ d\ \ d\ \ -b\ \ -d\ \ -a$	$d\ \ c\ \ d\ \ -c\ \ d\ \ -d\ \ b\ \ d\ \ a$
(5, 5, 10, 10)	$a\ \ c\ \ d\ \ a\ \ 0\ \ 0\ \ -a\ \ c\ \ d$	$a\ \ c\ \ d\ \ 0\ \ -c\ \ -d\ \ a\ \ -c\ \ -d$	
		$b\ \ c\ \ -d\ \ b\ \ 0\ \ 0\ \ -b\ \ c\ \ -d$	$b\ \ c\ \ -d\ \ 0\ \ -c\ \ d\ \ b\ \ -c\ \ d$
(8, 8, 8, 8)	$a\ \ a\ \ a\ \ -a\ \ b\ \ b\ \ -b\ \ b\ \ 0$	$b\ \ b\ \ b\ \ -b\ \ -a\ \ -a\ \ a\ \ -a\ \ 0$	
		$c\ \ c\ \ c\ \ -c\ \ d\ \ d\ \ -d\ \ d\ \ 0$	$d\ \ d\ \ d\ \ -d\ \ -c\ \ -c\ \ c\ \ -c\ \ 0$
(9, 9, 9, 9)	$a\ \ a\ \ a\ \ -a\ \ b\ \ b\ \ -b\ \ b\ \ c$	$b\ \ b\ \ b\ \ -b\ \ -a\ \ -a\ \ a\ \ -a\ \ -d$	
		$c\ \ c\ \ c\ \ -c\ \ d\ \ d\ \ -d\ \ d\ \ -a$	$d\ \ d\ \ d\ \ -d\ \ -c\ \ -c\ \ c\ \ -c\ \ b$

Appendix B (continued): Order 36 (Sequences with zero non-periodic autocorrelation function)

Design	A_1	A_2, A_3	A_4
(1, 26)	b −b b b a −b −b b −b	b −b −b −b −b b b −b b	
		b 0 0 0 0 0 b b 0	b 0 b 0 b b b −b 0
(2, 25)	b 0 −a 0 b 0 −b −b 0	b b −b −b −b −a b −b b	
		b 0 b b 0 b 0 b 0	b b −b b −b −b 0 b −b
(2, 31)	b −b b 0 a −b −b b b	−b b b 0 a b −b −b −b	
		−b b −b b 0 −b b b b	−b b b b b b −b b b
(3, 26)	−b b 0 b b 0 a b 0	−b −b a b −b b b 0 b	
		−b −b −b b 0 a −b 0 −b	0 −b b −b −b b b b −b
(3, 28)	b b b 0 a −b b b 0	b b b 0 −a −b b −b 0	
		b −b b −b a b −b −b −b	b −b −b b 0 −b b b b
(3, 30)	b b b 0 a −b b b −b	b −b b −b −b a b −b −b	
		b −b b −b 0 −a −b b b	b b b b b 0 −b −b b
(4, 25)	−b b b a 0 a −b −b b	b 0 b −a b a b 0 b	
		b 0 b b 0 −b −b 0 −b	−b b b −b 0 −b b b −b
(4, 27)	−b −b 0 a −b a b b b	b −b 0 −a 0 a b −b b	
		−b b b b −b −b b −b b	b b b 0 b 0 −b b b
(4, 29)	b b a b 0 −b a −b −b	b b −a b −b b a b b	
		b −b b b b 0 −b −b b	b −b b 0 −b −b b b −b
(5, 24)	−b a b 0 −b 0 −b 0 a	b 0 b b b 0 a −b b	
		a b −b b b 0 −b −a b	b b b −b 0 −b −b b −b
(5, 26)	a −b a b −b −b b b 0	b b a b −b −b 0 −b 0	
		b b 0 b 0 a b −a b	b −b b −b b b b −b −b
(5, 28)	−b −b b 0 a b −b a −b	b b −b b a b −b b b	
		b b −b 0 −a b b a −b	b b b b 0 −b b −b −b
(5, 29)	−b b b a 0 a −b −b b	b −b −b a −a −a −b b b	
		b −b b −b b b b b b	b b −b −b 0 −b b −b −b
(5, 31)	b b b b a −b −b −b b	−b b −b −b a b a −b b	
		b −b a b −a −b b −b b	−b b b b b −b b b b
(6, 23)	b a −b 0 −b 0 0 a −b	b a −b b a 0 −b −a b	
		b b 0 a 0 0 b −b b	b −b b b b b b −b −b
(6, 25)	a 0 b b −b 0 a −b −b	a 0 b a b 0 −a b b	
		−b b −b a b b −b −b −b	−b b −b b 0 −b b b b
(7, 15)	a 0 a −b 0 b 0 −b 0	b −b 0 −a −a a −b −b −b	
		a 0 0 −b 0 −b 0 0 0	b 0 −b 0 0 −b b a b
(7, 19)	−a −b a −b a −b 0 −b 0	a b −b b b 0 0 −b 0	
		b b −b −b b 0 0 a 0	a 0 b b a b −b −b 0

Appendix B (continued): Order 36 (Sequences with zero non-periodic autocorrelation function)

Design	A_1	A_2, A_3	A_4
(7, 22)	$a\ -b\ b\ 0\ a\ 0\ -b\ -b\ 0$	$b\ b\ -a\ b\ a\ b\ a\ b\ -b$	
		$b\ b\ 0\ -b\ 0\ a\ -b\ b\ 0$	$b\ -b\ -b\ -b\ 0\ b\ -b\ a\ b$
(7, 23)	$b\ 0\ a\ a\ -b\ b\ b\ b\ 0$	$-b\ -b\ 0\ a\ 0\ a\ -b\ b\ -b$	
		$0\ b\ a\ -b\ -a\ b\ -b\ b\ b$	$b\ b\ b\ -b\ b\ a\ -a\ -b\ -b$
(7, 24)	$b\ a\ b\ b\ 0\ a\ b\ -b\ -b$	$a\ -b\ 0\ a\ b\ -b\ b\ -b\ -b$	
		$b\ b\ -b\ -b\ -b\ b\ b\ -b\ b$	$b\ b\ 0\ b\ -a\ b\ -a\ a\ a$
(7, 26)	$b\ a\ b\ a\ 0\ -a\ b\ -b\ -b$	$b\ -b\ 0\ a\ b\ b\ -b\ b\ b$	
		$b\ -b\ -b\ -b\ a\ -b\ b\ -b\ -b$	$b\ b\ -b\ a\ -b\ -b\ 0\ a\ b$
(8, 19)	$a\ b\ 0\ a\ b\ -b\ 0\ 0\ 0$	$b\ -a\ -b\ -b\ b\ -a\ b\ 0\ 0$	
		$b\ b\ -a\ -b\ -a\ a\ a\ -b\ b$	$b\ b\ b\ 0\ b\ 0\ 0\ b\ -b$
(8, 21)	$b\ 0\ 0\ a\ b\ 0\ a\ 0\ 0$	$b\ -b\ a\ -b\ -b\ -b\ a\ b\ 0$	
		$b\ -b\ b\ b\ b\ b\ -b\ -b\ b$	$b\ 0\ a\ b\ a\ -a\ -a\ b\ -b$
(8, 23)	$b\ 0\ a\ a\ -b\ b\ b\ b\ 0$	$b\ b\ 0\ -a\ 0\ -a\ b\ -b\ b$	
		$b\ b\ -b\ b\ -a\ -b\ a\ b\ 0$	$b\ b\ b\ -b\ b\ a\ -a\ -b\ -b$
(8, 25)	$b\ a\ b\ b\ 0\ a\ b\ -b\ -b$	$a\ -b\ 0\ a\ b\ -b\ b\ -b\ -b$	
		$b\ b\ -b\ -b\ -b\ b\ b\ -b\ b$	$b\ b\ 0\ b\ -a\ b\ -a\ a\ a$
(9, 20)	$a\ a\ -b\ b\ a\ b\ 0\ 0\ -b$	$b\ -b\ 0\ b\ b\ b\ b\ b\ -b$	
		$b\ -b\ 0\ b\ a\ -a\ 0\ 0\ b$	$b\ b\ -b\ 0\ -a\ -a\ a\ -b\ a$
(9, 21)	$b\ b\ a\ b\ -b\ -b\ a\ 0\ -b$	$b\ -b\ a\ 0\ 0\ a\ -b\ b\ b$	
		$b\ b\ -b\ 0\ 0\ b\ -a\ b\ -b$	$b\ b\ a\ b\ a\ -a\ -a\ 0\ b$
(9, 22)	$b\ b\ 0\ 0\ a\ a\ b\ -b\ b$	$b\ b\ -a\ 0\ 0\ -a\ b\ -b\ b$	
		$b\ b\ b\ a\ -b\ -b\ -a\ a\ -b$	$b\ a\ 0\ b\ -b\ -a\ b\ b\ -b$
(9, 24)	$b\ -a\ b\ 0\ -a\ b\ b\ -b\ -b$	$-b\ -b\ a\ b\ b\ 0\ a\ b\ -b$	
		$b\ b\ a\ b\ b\ -b\ b\ -b\ b$	$b\ -b\ -b\ a\ 0\ a\ -a\ -a\ -b$
(10, 21)	$b\ -b\ 0\ b\ b\ b\ 0\ a\ b$	$b\ b\ -b\ a\ -b\ -b\ -b\ b\ -b$	
		$b\ -b\ -a\ -a\ 0\ a\ -a\ b\ 0$	$b\ -b\ -b\ a\ a\ b\ a\ -a\ 0$
(10, 23)	$b\ b\ 0\ -a\ a\ -a\ b\ a\ a$	$b\ b\ -a\ -b\ b\ -b\ 0\ -a\ -a$	
		$b\ a\ -b\ b\ 0\ b\ -b\ -a\ b$	$b\ -b\ b\ b\ b\ b\ b\ -b\ -b$
(11, 18)	$b\ b\ a\ -b\ a\ -b\ a\ b\ -b$	$b\ 0\ b\ 0\ 0\ a\ 0\ 0\ b$	
		$b\ a\ -b\ 0\ -b\ a\ -b\ -a\ -b$	$b\ a\ -b\ -a\ 0\ -a\ -b\ a\ b$
(11, 19)	$b\ a\ b\ -b\ -b\ a\ a\ -b\ b$	$b\ 0\ b\ a\ -b\ b\ b\ b\ -b$	
		$b\ a\ -a\ b\ -a\ b\ 0\ 0\ 0$	$b\ 0\ a\ a\ -a\ -b\ -b\ -a\ 0$
(11, 22)	$b\ -b\ -b\ b\ b\ b\ 0\ b\ a$	$b\ -b\ -b\ -b\ 0\ a\ -b\ a\ -a$	
		$b\ -b\ b\ -b\ -b\ -a\ -a\ -a\ b$	$b\ b\ 0\ a\ -a\ -a\ a\ b\ -b$
(11, 23)	$b\ -b\ -b\ -b\ a\ b\ a\ -b\ a$	$b\ 0\ b\ -b\ b\ a\ -b\ b\ b$	
		$a\ -b\ -a\ b\ -b\ -a\ -b\ 0\ -b$	$b\ b\ a\ -a\ -a\ -b\ -b\ a\ -b$

Appendix B (continued): Order 36 (Sequences with zero non-periodic autocorrelation function)

Design	A_1									A_2, A_3									A_4								
(11, 24)	a	a	$-b$	$-b$	a	b	$-b$	b	$-b$	b	$-a$	b	$-b$	a	b	a	b	b									
										a	b	a	$-a$	$-a$	a	$-b$	b	b	b	b	b	$-b$	0	b	$-b$	$-b$	$-b$
(12, 17)	a	0	a	b	0	0	b	0	0	b	a	$-b$	b	$-b$	$-b$	$-b$	a	0									
										a	0	a	$-a$	$-b$	a	$-b$	b	b	b	b	a	$-b$	b	a	b	$-a$	$-a$
(12, 19)	b	b	a	$-b$	a	$-b$	0	b	$-b$	b	a	b	0	b	0	b	a	$-b$									
										b	b	0	$-a$	0	b	$-a$	$-b$	b	b	$-a$	a	$-a$	$-a$	$-b$	a	a	b
(12, 22)	b	a	$-b$	0	$-b$	a	b	b	$-b$	b	b	a	b	a	b	$-b$	$-b$	b									
										b	$-b$	a	a	$-b$	$-b$	$-b$	a	$-a$	b	$-b$	a	$-b$	$-a$	0	$-a$	$-b$	a
(12, 24)	b	$-b$	a	$-b$	$-b$	$-b$	a	b	a	b	b	$-b$	b	a	b	b	b	$-b$									
										b	$-b$	$-b$	b	$-b$	$-b$	$-b$	$-a$	b	a	$-a$	a	a	$-a$	$-a$	$-a$	b	$-b$
(13, 14)	a	a	a	$-a$	$-a$	a	$-a$	0	0	b	b	0	0	$-a$	0	b	0	$-a$									
										b	$-b$	b	a	$-b$	a	b	b	0	b	b	a	0	b	$-b$	0	$-b$	a
(13, 17)	a	a	0	a	$-b$	b	$-b$	$-b$	0	a	b	b	b	$-b$	0	a	b	0									
										a	$-a$	b	b	b	$-b$	b	0	$-b$	b	0	$-a$	$-a$	$-b$	a	a	$-a$	a
(13, 18)	b	$-b$	b	b	b	a	b	a	$-b$	b	$-a$	$-b$	b	b	$-a$	b	$-b$	b									
										a	$-b$	$-b$	0	b	b	a	0	0	a	$-a$	a	a	$-a$	$-a$	$-a$	0	0
(13, 20)	b	b	$-b$	0	$-a$	$-a$	b	$-b$	b	b	$-b$	b	0	a	a	b	$-b$	$-b$									
										a	$-b$	$-b$	a	0	$-a$	b	b	a	a	b	b	a	$-a$	a	b	b	$-a$
(13, 21)	b	a	0	$-a$	b	$-a$	b	a	$-b$	b	b	b	a	b	a	$-b$	$-b$	b									
										a	b	$-b$	$-b$	a	$-b$	b	$-b$	a	a	b	a	b	$-a$	$-b$	0	b	$-a$
(13, 23)	b	a	$-b$	b	$-b$	$-b$	$-b$	a	a	b	b	$-b$	b	b	b	a	$-b$	a									
										b	b	$-b$	b	$-b$	a	b	b	$-a$	a	a	$-a$	b	a	$-a$	$-b$	$-a$	$-b$
(14, 19)	a	$-b$	$-b$	b	b	0	a	b	a	b	$-b$	a	$-b$	$-b$	a	a	$-b$	$-a$									
										b	$-b$	b	$-b$	0	b	b	b	a	a	a	b	$-a$	$-b$	0	$-a$	a	$-a$
(14, 20)	b	$-b$	$-a$	0	$-a$	$-b$	$-a$	b	$-b$	a	$-b$	$-b$	$-b$	$-a$	b	a	a	$-b$									
										b	$-b$	$-b$	0	$-b$	$-b$	$-a$	$-b$	b	b	$-a$	$-a$	a	b	$-a$	a	$-b$	a
(14, 21)	b	$-a$	$-a$	$-b$	b	$-b$	$-a$	b	b	b	a	b	$-a$	b	a	$-b$	a	$-b$									
										b	$-a$	a	b	b	b	a	$-b$	b	b	$-a$	$-b$	$-a$	0	a	b	a	$-b$
(14, 22)	a	b	a	$-b$	b	$-b$	$-b$	$-b$	a	b	b	$-b$	$-b$	$-a$	$-b$	b	$-b$	$-a$									
										a	b	b	a	$-b$	b	b	$-a$	b	a	$-a$	$-b$	a	a	b	$-a$	b	$-a$
(15, 18)	a	a	$-a$	a	a	a	0	$-b$	$-a$	a	a	0	$-b$	$-a$	a	$-a$	b	a									
										a	$-b$	b	b	b	b	b	$-b$	b	b	$-b$	b	b	0	$-b$	$-b$	b	$-a$
(15, 19)	b	a	$-b$	$-b$	b	0	a	a	$-b$	b	$-b$	a	$-a$	$-b$	$-a$	$-a$	b	b									
										a	b	$-b$	b	b	b	$-a$	b	a	a	b	a	0	b	$-a$	a	$-b$	$-a$
(15, 21)	b	$-a$	$-b$	b	$-a$	$-b$	b	$-a$	b	b	$-b$	$-b$	$-b$	b	b	a	b	a									
										b	$-b$	$-b$	$-b$	a	$-b$	$-a$	$-b$	$-a$	b	$-a$	$-a$	a	a	a	$-a$	a	$-b$