Semiconductor optical amplifiers in negative-exponential fading: regenerators and pre-amplifiers

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Abstract: In this study, the authors discuss the mitigation of negative-exponential fading in optical wireless communication systems. The mitigation technique involves the utilisation of a semiconductor optical amplifier (SOA) that, depending on the link arrangement, acts either as regenerator or pre-amplifier. As a regenerator, the SOA gain saturates during normal link operation and increases when the link experiences a fade. This unbalanced SOA operation serves towards the equalisation of the signal power at its output and fades become less severe and of reduced duration. The analytical results predict that the fade probability is reduced by over 90% and the scintillation index is improved by 75% for an optimal level of the received power. Moreover, the average duration of fades is also reduced by 68% for the same power level. As a pre-amplifier, the SOA modifies the noise statistics at the receiver and provides a static sensitivity increase of at least 10 dB at 10 Gb/s, depending on the bit-error-rate (BER) target. The analytical results show that this sensitivity improvement imparts a reduction of one order of magnitude on the average BER, of at least 94% on the outage probability and of at least 78% on the average duration of fades.

1 Introduction

The mitigation of fades is an important topic in outdoor optical wireless (OW) systems, where atmospheric turbulence induces time-varying changes to the refractive index, which in turn affect the amplitude, phase and propagation direction of the optical signal [1–3]. These changes ultimately manifest as time-varying power fluctuations of the received signal and when the turbulence is intense enough, the received power decreases below the receiver sensitivity and the link is lost. This corresponds to a fade event, which affects the optical wireless communication (OWC) system both in terms of capacity, as well as latency. Owing to the negative impact of fades on the OWC systems performance, a number of techniques have been proposed for decreasing the fade probability and the average fade duration (AFD), and therefore, for minimising their detrimental effect. In weak (log-normal) fading conditions it is typically sufficient to utilise an aperture-averaging technique, where a large-aperture receiver collects stray light [4, 5] or beam focusing [6]. The mitigation of more intense (gamma–gamma) fading, however, requires spatial or temporal diversity techniques [7–14]. Alternatively, more advance mitigation techniques can be implemented using aperture averaging and diversity in conjunction with relaying [15], amplification [16–18] and/or coding schemes [19–23], which essentially adapt the transmission rate with the channel capacity.

Within the context of fade mitigation, we have previously reported and analysed the utilisation of semiconductor optical amplifiers (SOAs) in outdoor OWC systems in two distinct modes of operation. Both modes, re-generation and pre-amplification, rely on utilising a SOA before opto-electronic conversion in a setup that is illustrated in Fig. 1. Under partial saturation, the SOA can act as a regenerator that mitigates fades by providing unbalanced gain to the incoming OW signal [18]. If the link is in a fade state, then the SOA is not saturated by the received signal and provides linear gain, whereas the received signal saturates the SOA and experiences limited or no gain at all during the absence of fades. The gain saturation effect provides increased amplification to the fade impaired segments of the signal and, as a result, the signal power fluctuations become less intense at the SOA output [18, 24]. On the other hand, if the average received optical power is not sufficient to saturate the SOA, then the device acts as an almost linear amplifier regardless of the channel state. In this mode of operation, a second additional benefit can be obtained from the improvement that is observed at the receiver sensitivity because of the signal and noise beating process on the photo-diode [25, 26]. In [27], we demonstrated that the sensitivity improvement translates directly into a link gain that drastically improves the OWC system performance in terms of the attained bit-error-rate (BER).

Although previous works have demonstrated the potential of SOA-based fade mitigation, the fading conditions that were considered correspond to either weak (log-normal) or moderate (gamma–gamma) turbulence. It is still of interest, however, to investigate the performance of the proposed technique in more adverse (saturated) fading conditions that appear over increased OWC link lengths and/or intense turbulence. In this regime, the received power fluctuations are typically considered to follow a negative-exponential distribution [28–33], which corresponds to the propagation of light beams through a very large number of scattering cells [34]. The primary goal of this work is to demonstrate the applicability of SOA-based regeneration and pre-amplification in this intense fading environment and discuss in a quantitative fashion the performance improvement that can be achieved. To this end, we first derive in Section 2 an analytical model for calculating the first- and second-order statistics of the SOA-based regenerator. We then utilise the presented analytical model to assess restorative properties of the SOA and demonstrate that a significant reduction of over 90 and 68% can be expected for the fade probability and duration, respectively. In addition, we investigate the quality of the signal in terms of the scintillation index (SI) and show that it can also be reduced by 75% at maximum. As far as the SOA pre-amplifier is concerned, we derive in Section 3 a mathematical framework that evaluates the system performance in terms of the BER that is attainable at each channel state. The model takes into account all noise variances that arise from the signal and noise beating process and associates them with the channel state, thus enabling the treatment of the system BER as a random variable whose values are determined by the negative-exponential statistics. We then utilise the presented…
framework to analytically calculate metrics, such as the average BER, the outage probability and the AFD of the SOA-assisted system. The analytical results demonstrate that the SOA improves the average BER by one order of magnitude, the outage probability by over 94% and the average duration of fades by over 78% for practical receiver arrangements. Finally, we shortly discuss the impact of background noise on the pre-amplifier performance.

2 SOA as regenerator

In the current section, we first analytically calculate the probability density function (pdf) of the signal at the SOA output, which will enable the derivation of the fade probability and the SI. Both are the two most commonly considered first-order metrics that describe the signal quality in the presence of fades. Our goal is to demonstrate that the deployment of the SOA and its unbalanced gain saturation results in a signal that, although not fully restored, is of superior quality when compared with the original negative-exponential impaired one, and as a result the SOA essentially acts as a regenerator. The fade mitigation capabilities of the SOA-based equaliser are also demonstrated by the second-order statistics of the signal at the output of the SOA. Even though the SOA drastically reduces the fade probability, as we will show next, fades are still expected to occur and it is of practical importance to have an estimation of the AFD. The AFD affects the OWC system latency and its evaluation is particularly useful when considering temporal diversity techniques, since it provides an estimate of the required buffering, as well as when designing link layer automatic request protocols in which the frame and window sizes should be tailored so that a full-window transmission exceeds the fade duration.

2.1 SOA output power pdf

We begin our analysis by considering that the average power $P_{\text{in}}$ of each received pulse at the SOA input follows the channel response. Under strong fading conditions $P_{\text{in}}$ can be modelled as a negative-exponential random variable [34] with a pdf equal to

$$f_{P_{\text{in}}}(z) = \frac{1}{\bar{P}_{\text{in}}} \exp \left( - \frac{z}{\bar{P}_{\text{in}}} \right)$$

(1)

where $\bar{P}_{\text{in}}$ is the average received optical power. The received pulses are driven to the SOA and each pulse experiences a gain that is dependent on its energy. We assume that the gain recovery time is limited to less than one bit period $T_b$, so that the SOA fully recovers to its small signal $G$ gain after each incoming pulse. The input $U_{\text{in}}$ and output $U_{\text{out}}$ pulse energies obey [35, 36]

$$U_{\text{out}} = U_{\text{sat}} \log \left[ 1 + G \left( \exp \left( \frac{U_{\text{in}}}{U_{\text{sat}}} \right) - 1 \right) \right]$$

(2)

with $U_{\text{sat}}$ being the SOA saturation energy. Input and output pulse energies are proportionally related to average optical powers

$$U_{\text{in}} = P_{\text{in}} T_b$$

(3a)

and

$$U_{\text{out}} = P_{\text{out}} T_b$$

(3b)

and as a result (2) can be re-written as

$$P_{\text{out}} = U_{\text{sat}} \log \left[ 1 + G \left( \exp \left( \frac{P_{\text{in}}}{P_{\text{sat}}} \right) - 1 \right) \right]$$

(4)

where $P_{\text{sat}}$ is the rate-dependent saturation parameter of the SOA

$$P_{\text{sat}} = \frac{U_{\text{sat}}}{T_b}$$

(5)

The pdf of the optical signal envelope at the output of the SOA is calculated by combining (1) and (4)

$$f_{P_{\text{out}}}(z) = \frac{1}{GP_{\text{in}}} \exp \left( \frac{z}{P_{\text{sat}}} \right) \left( T \left( \frac{z}{P_{\text{sat}}} \right) \right)^{-(P_{\text{sat}}/P_{\text{in}})^{-1}}$$

(6)

with $T(x) = [G - 1 + \exp (x)]/G$. For discussion purposes, we further define the normalisation parameters $r$ (normalised input power) and $u$ (normalised output power)

$$r = \frac{\bar{P}_{\text{in}}}{P_{\text{sat}}}$$

(7a)

and

$$u = \frac{P_{\text{out}}}{P_{\text{in}}}$$

(7b)

to obtain the normalised SOA output power pdf as

$$f_u(z) = \frac{1}{G} \exp \left( rz \right) \left( T (rz) \right)^{-(1/r)^{-1}}$$

(8)
2.2 First-order statistics

A fade occurs whenever the output power $u$ remains below a predetermined threshold $u_t$, and by integrating (8), we find that the fade probability equals to

$$P_{\text{out}} = \Pr [u < u_t] = \int_0^{u_t} f_u(z) \, dz = 1 - \left( T(\rho u_t) \right)^{-1/r} \quad (9)$$

The fade probability is plotted in Fig. 2 against the normalised receiver power threshold $\rho$

$$\rho = \frac{u_t}{u_{\text{rms}}} \quad (10)$$

where $u_{\text{rms}}$ is numerically evaluated using

$$u_{\text{rms}}^2 = \frac{1}{G} \int_0^{\infty} z^2 \exp \left( rz \right) \left( T(\rho z) \right)^{-1/r} \, dz \quad (11)$$

This normalisation is performed so as to take into account the average static gain which is provided by the SOA, and thus, compare pdfs that correspond to different average output powers in a fair manner.

Fig. 2 shows that the fade probability is considerably decreased when the SOA is deployed in comparison with the incoming negative-exponential signal. A fade probability decrease of over 90% is predicted for a 20 dB small-signal gain SOA, whereas a 30 dB gain device further reduces the fade probability by 95%. This is expected from the SOA response as detailed in (2), since higher gain devices lead to better equalisation of the output pulse energies. The results also suggest that increasing the average input power $r$ is always beneficial, still the attained benefit saturates and input powers of over 1 do not yield a significant restorative improvement. This last observation can be further explored by numerically evaluating the SI of the signal at the SOA output

$$\sigma_u^2 = \frac{\sqrt{u_{\text{rms}}^2 - \bar{u}^2}}{\bar{u}} \quad (12)$$

where

$$\bar{u} = \frac{1}{G} \int_0^{\infty} z \exp \left( rz \right) \left( T(\rho z) \right)^{-1/r} \, dz \quad (13)$$

The SI is plotted in Fig. 3 against the normalised input power $r$ for SOAs with 20 and 30 dB small-signal gains. The figure shows that the output signal SI values are significantly lower than those of the original negative-exponential (SI = 1), which serves to verify the beneficial impact of the equalisation process. Similar to the fade probability results, high gain SOAs are also capable of attaining lower SIs as compared to lower gain ones and are therefore preferable. Finally, it is noteworthy to mention that the SI is minimised for input powers approximately equal to the saturation parameter $P_{\text{sat}}$ of the SOA. This is expected from the SOA operation, since low average input powers lead to a non-saturated device that always provides linear gain, thus no equalisation, to both fade impaired and non-impaired pulses. In the totally opposite regime, a fully saturated device will provide unity gain to all pulses irrespective of their energy and as a result fades are not equalised, as well. The optimal point of operation corresponds to a partially saturated SOA ($r \approx 1.0$) that provides increasing gain to weaker pulses and thus restores the input signal to some extent.

2.3 Second-order statistics

The AFD at the output of the SOA is calculated from the output signal level crossing rate (LCR), which in turn requires the knowledge of the joint pdf between the output signal and its time derivative. The joint pdf at the SOA output is calculated from the
The joint pdf of the exponentially faded input signal, given by [37]

\[ f_{u,v}(z, w) = f_z(z) \frac{1}{\sqrt{4\pi P_{\text{sat}}(0)}} \exp\left(-\frac{w^2}{4z P_{\text{sat}}(0)}\right) \]  

where

\[ f_z(z) = \exp(-z) \]  

\( r(t) \) is the normalised covariance function that describes the rapidity of fading and the dot operator corresponds to the time derivative. To calculate the joint pdf at the SOA output we utilise (4) to obtain the relations between the input \( v \) and output \( u \) signals and their derivatives, resulting in

\[ v = \left( \frac{2}{\pi} \right) \log \frac{G - 1 + \exp(r u)}{G} \]  

\[ \dot{v} = \frac{\exp(r u)}{G - 1 + \exp(r u)} \]  

After applying the variable transform of (16) into (14), we find that the joint pdf at the SOA equals

\[ f_{u,v}(z, w) = f_z(z) \frac{\exp(-w^2/2\sigma_{\text{SOA}}^2)}{\sqrt{2\pi \sigma_{\text{SOA}}}} \]  

with variance \( \sigma_{\text{SOA}}^2 \) being defined as

\[ \sigma_{\text{SOA}}^2 = \frac{2P_{\text{ase}} P_{\text{sat}}(0)(G - 1 + \exp(z/P_{\text{sat}})^2) \log[T(z/P_{\text{sat}})]}{\exp(2z/P_{\text{sat}})} \]  

By integrating (17) with respect to the time derivative \( w \) and keeping the normalisation definitions of (7), we calculate the LCR at the output of the SOA

\[ \text{LCR}(u_r) = \int_0^\infty f_{u,v}(u_r, w) dw \]  

\[ = \frac{\exp\left(\frac{1}{2} \log[T(u_r)]\right)}{\left(\log[T(u_r)]\right)^{1/2}} \]  

and the AFD is finally derived after combining the fade probability and the LCR as

\[ \text{AFD}(u_r) = \sqrt{\frac{\pi}{r(0)}} \left(\frac{1}{1/r} \log[T(u_r)]\right)^{1/2} - 1 \]  

Equation (20) is plotted in Fig. 4 against the normalised threshold \( \rho \). The figure suggests that the 20 dB-gain SOA reduces the duration of fades by 68–84%, whereas the 30 dB-gain SOA achieves an even better reduction of 87–94%. As a result the duration of fades, and therefore the system latency, can be decreased by up to two orders of magnitude by utilising the SOA-based SOA, provided that the SOA is saturated to the appropriate level. Similar to the first-order statistics, a signal power approximately equal to the saturation parameter is adequate to attain maximum AFD reduction.

### 3 SOA as pre-amplifier

In this section, we first derive the mathematical model that will enable the BER assessment the pre-amplified OWC system. To this end, we briefly detail the signal and noise properties of the SOA output and then correlate the system BER with the OWC channel state. We further discuss the performance improvement that is introduced by the SOA in the first-order statistics of the OWC system, and in particular the average BER and the outage probability, which provide a measure of the GW link reliability. Apart from these metrics, it is equally important to evaluate the second-order statistics of the pre-amplified system. Therefore, similarly to the regenerator analysis, we focus on the analysis for the AFD that is defined as the average duration that the OWC system remains below a desired BER level after a fade event.

#### 3.1 Noise model and BER calculation

We first consider that the SOA generates an optical noise component because of the amplified spontaneous emission (ASE), which is described by the ASE spectral density

\[ P_n = n_p h c \frac{\lambda}{\alpha} \]  

where \( c \) is the vacuum light speed, \( h \) is the Planck constant, \( n_p \) is the population inversion factor and \( \lambda \) denotes the wavelength. The optical signal and the ASE beat on the square-law detector of the receiver (photodiode) and as a result several electrical noise components appear at the photodiode output. The associated noise variances are denoted as thermal, shot, signal-spontaneous beating
Table 1 System parameters

<table>
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<th>Parameter</th>
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<th>Value</th>
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<tr>
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<td>$\lambda$</td>
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</tr>
<tr>
<td>line rate</td>
<td>$T_c$</td>
<td>10 Gb/s</td>
</tr>
<tr>
<td>SOA small-signal gain</td>
<td>$G$</td>
<td>20 or 30 dB</td>
</tr>
<tr>
<td>saturation parameter</td>
<td>$P_{sat}$</td>
<td>1 mW</td>
</tr>
<tr>
<td>population inversion factor</td>
<td>$n_0$</td>
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<td>photodiode responsivity</td>
<td>$R$</td>
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<tr>
<td>receiver temperature</td>
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</tr>
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</tr>
<tr>
<td>optical bandwidth</td>
<td>$B_o$</td>
<td>50 GHz</td>
</tr>
</tbody>
</table>

and spontaneous–spontaneous beating, and are calculated as

$$\sigma_{th}^2 = \frac{4k_B TF_o B_e}{R_L}$$  \hspace{1cm} (22a)

$$\sigma_{sp-out}(z) = 2qR(P_{sat}(z) + (G - 1)P_{sat})B_e$$  \hspace{1cm} (22b)

$$\sigma_{sp-sp}(z) = 4R^2P_{sat}(z)(G - 1)P_{sat}B_e$$  \hspace{1cm} (22c)

and

$$\sigma_{sp-sp}^2 = R^2((G - 1)P_{sat})^2(2B_o - B_e)B_e$$  \hspace{1cm} (22d)

respectively. In (22), $B_e$ and $B_o$ are the electrical and optical bandwidths, respectively, $R$ is the photodiode responsivity, $T$ is the receiver temperature, $k_B$ denotes the Boltzmann constant, $F_o$ is the electric noise figure and $R_L$ is the resistor load. All respective parameters and the values that are used for the presentation are summarised in Table 1.

Given (4) and (22), the signal and noise powers in the ‘1’ and ‘0’ symbols are directly calculated as

$$I_0(z) = RP_{sat}(z)$$  \hspace{1cm} (23a)

$$\sigma_0^2(z) = \sigma_{th}^2 + \sigma_{sp-out}^2 + \sigma_{sp-sp}^2$$  \hspace{1cm} (23b)

and

$$I_0 = 0$$  \hspace{1cm} (24a)

$$\sigma_0^2 = \sigma_{th}^2 + \sigma_{sp-out}(0) + \sigma_{sp-sp}$$  \hspace{1cm} (24b)

respectively. Following the above, the BER performance of the system is evaluated by

$$BER(z) = \frac{1}{2}\erfc\left(\frac{Q(z)}{\sqrt{2}}\right)$$  \hspace{1cm} (25)

with

$$Q(z) = \frac{I_1(z)}{\sigma_0 + \sigma_1(z)}$$  \hspace{1cm} (26)

In (25), it is assumed that the receiver is capable of estimating the Channel State Information (CSI-capable) and setting its decision threshold on a symbol-by-symbol fashion to [38]

$$I_{th}(z) = \frac{\sigma_0 I_0(z)}{\sigma_0 + \sigma_1(z)}$$  \hspace{1cm} (27)

3.2 First-order statistics

The average BER is calculated after integrating the BER variable over all possible negative-exponential channel states following

$$BER = \frac{1}{2}\int_0^{\infty}\erfc\left(\frac{Q(z)}{\sqrt{2}}\right)f_{P_{in}}(z)dz$$  \hspace{1cm} (28)

The integral is evaluated in a numerical fashion from the values of Table 1 and the results are plotted in Fig. 5 for two SOAs with 20 and 30 dB small signal gains. The figure clearly demonstrates a very significant improvement of the average BER of the system with the SOA and a 16 dB gain in the link budget is observed at any fixed BER level. This is expected from the sensitivity increase that is brought about by the SOA, which equals 14.3 dB at a BER of $10^{-3}$ and 12.5–13.8 dB at a BER of $10^{-6}$ given the numerical values under consideration. Moreover, the deployment of the SOA improves the average BER at least one order of magnitude for the presented input powers, thus the SOA adds to the reliability of the system and can contribute towards lowering the fade margin that is required in a practical system.

The exact contribution of the SOA to the link margin, however, can only be explored via the outage probability, since real-world systems are designed with a specific maximum acceptable BER level in mind, and additional techniques (mainly forward error correction) are introduced to recover from link errors. In these systems, an outage occurs whenever the BER at the receiver remains below the pre-defined target level $BER_0$ and following the analysis presented in the previous section the outage probability is calculated by

$$P_{out} = Pr\{BER(z) > BER_0\}$$  \hspace{1cm} (29)

or equivalently by

$$P_{out} = Pr\{z \leq P_{in}\} = \int_0^{P_{in}} f_{P_{in}}(z)dz = 1 - \exp\left(-\frac{P_{in}}{P_{in}}\right)$$  \hspace{1cm} (30)

where $P_{in}$ is the receiver sensitivity that is required to achieve $BER_0$. The equivalence of (29) and (30) is justified from the fact that $BER_0$ is exceeded whenever the input power remains below the corresponding sensitivity $P_{in}$. Furthermore, the corresponding receiver sensitivity is obtained after numerically solving

$$BER(P_{in}) = \frac{1}{2}\erfc\left(\frac{Q(P_{in})}{\sqrt{2}}\right) = BER_0$$  \hspace{1cm} (31)
3.3 Second-order statistics

The AFD is calculated from the outage probability (29) and the LCR of the OWC link. For negative-exponential fading the LCR (normalised over the maximum Doppler frequency shift) is given by

$$LCR(P_a) = \exp\left( -\frac{P_a}{P_m} \right) \sqrt{\frac{P_m}{P_a}}$$

(32)

As we detail in [27], it is sufficient for the pre-amplified system to evaluate the LCR at the receiver sensitivity $P_a$ that is the required to attain the desired BER level $BER_0$ following (31). This result simplifies the calculation of the AFD, which is obtained in a straight-forward manner as

$$AFD(P_a) = \frac{\Pr\{ z \leq P_a \}}{LCR(P_a)} = \frac{\exp(P_a/P_m) - 1}{\sqrt{(P_a/P_m)}}$$

(33)

Equation (33) is plotted in Fig. 7 for a system with maximum BER level of $10^{-3}$ and $10^{-5}$. It can be verified from the figure that the SOA has a significant impact on the OW system AFD, owing to the expected sensitivity improvement. As a result, the link margin of 14.3 dB for the BER of $10^{-3}$ and 12.5–13.8 dB for the BER of $10^{-5}$, that was calculated in the outage probability analysis is also valid for the AFD analysis. With respect to the AFD itself, an AFD improvement of over 80% is predicted for the $10^{-3}$ BER level, whereas the corresponding improvement is marginally reduced to 78% for the $10^{-5}$ BER level.

Following the above, the SOA is well capable of drastically reducing both the probability of a fade and its average duration thus enabling a more reliable and lower latency outdoor OW system. In addition, the same link margin that is predicted for the first-order statistics analysis is also observed for the AFD and as a result the SOA-assisted receiver performance can be thoroughly described in a systematic fashion by the required sensitivity improvement and the statistics that govern fading.

3.4 Impact of the background noise

A final aspect that is interesting to address with respect to the pre-amplifier mode relates to the impact of the background noise. Despite the fact that the background noise is generally lower in the 1550 nm window as compared with the 850 nm one, since the solar radiation reduces at increasing wavelengths, any background noise that is coupled to the SOA receives a significant impact on the OW system AFD, owing to the SOA small signal gain. With respect to the pre-amplifier gain, the SOA-assisted system exhibits a reduced outage probability for all practically anticipated power levels. The reduction amounts to 96% for the $10^{-3}$ and over 94% for the $10^{-5}$ BER level, and exceeds the 90% reduction that is required to increase the system percentile availability by one additional ‘9’.

$$\sigma_{\text{out}}^2(\omega(z)) = 2\sigma R(\overline{P_{\text{out}}}(\omega(z)) + \left( (G - 1)P_n + GP_{bg} \right)B_e B_c)$$

(34a)

$$\sigma_{\text{bg-sp}}^2(\omega(z)) = 4R^2\overline{P_{\text{out}}}(\omega(z))\left( (G - 1)P_n + GP_{bg} \right)B_e$$

(34b)

and

$$\sigma_{\text{bg-sp}}^2 = R^2\left( (G - 1)P_n + GP_{bg} \right)^2(2B_e - B_c)B_c$$

(34c)

Again, a worst-case scenario is considered for the system operation in (34), and the background radiation does not saturate the SOA gain and receives small-signal gain amplification.
The exact power level of $P_{bg}$ is calculated by parameters that include the environmental conditions and the field of view of the receiver, following a more detailed analysis that can be found elsewhere [40]. It is not the absolute value of $P_{bg}$, however, that plays an important role in the pre-amplifier operation; rather it is the relative magnitude of $P_{bg}$ with respect to $P_n$ that proves critical, since both noise powers receive approximately the same gain $G$. This is presented in Fig. 8, which plots the average BER performance of the SOA-assisted system for the cases of (a) zero level background noise, (b) equal powers of background and ASE noise and (c) background noise power ten times stronger than the ASE.

Consequently, the introduced power penalty is approximately equal to the increase in the optical noise. As far as the 30 dB gain SOA is concerned, the power penalty is higher since both the signal-noise and noise-noise beating terms must be taken into account, especially when the background radiation dominates over the ASE. In contrast, the original non-amplified OWC system presents a totally different behaviour and a very limited power penalty, less than 0.5 dB, is observed when background noise is present. This is also theoretically expected, since the thermal noise dominates the receiver and the $Q$-factor is relatively insensitive to the noise components that correspond to the background radiation.

Following the above, the deployment of the pre-amplifier is only beneficial when the background noise can be kept lower than or comparable with the ASE.

4 Conclusions

We have evaluated in an analytical fashion the performance of a SOA-assisted outdoor OWC link in the negative-exponential fading regime. To this end, we provided a mathematical framework that fully describes the SOA operation both as a regenerator and pre-amplifier and derived analytical results for key first- and second-order statistics of the system, including the SI, outage probability, average BER and fade duration. The presented analytical results clearly demonstrate that all the metrics under consideration are improved by a sizable percentage and, consequently, both modes of operation are applicable, although different saturation levels are optimising the SOA performance in each mode.
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6 References