Appl. Num. Anal. Comp. Math. **1**, No. 2, 547 – 551 (2004) / **DOI** 10.1002/anac.200410018

Optimal Ship Routing Based on Wind and Wave Forecasts

D.S. Vlachos*

Hellenic Center for Marine Research, PO BOX 712, 19013, Anavyssos, Greece

Received 5 October 2004, revised 3 December 2004, accepted 3 December 2004
Published online 20 December 2004

Short term accurate wind and wave forecasts can be used for optimal ship routing. In this work, a method is presented for manipulating the POSEIDON [1] system forecasts for the Greek seas and for producing optimal routes for small and medium size ships. The method is composed from a geometrical calculation of all possible routes between two points and from an iterative algorithm which approximates the optimal (or an almost optimal) route.

1 Introduction

Optimization of ship routing is closely related to both ship characteristics and environmental factors. Ship and cargo characteristics have a significant influence on the application of ship routing. Ship size, speed capability and type of cargo are important considerations in the route selection process prior to sailing and the surveillance procedure while underway. A ship's characteristics identify its vulnerability to adverse conditions and its ability to avoid them. Ship performance curves (speed curves) are used to estimate the ship's speed of advance while transiting the forecast sea states. The curves indicate the effect of head, beam and following seas of various significant wave heights on the ship's speed.

Environmental factors of importance to ship routing are those elements of the atmosphere and ocean that may produce a change in the status of a ship transit. In ship routing, consideration is given to wind, seas, fog and ocean currents. While all of the environmental factors are important for route selection and surveillance, optimum routing is normally considered attained if the effects of wind and seas can be optimized. The effect of wind speed on ship performance is difficult to determine. In light winds (less than 20 knots) ships lose speed in headwinds and gain speed slightly in following winds. For higher speeds, ship speed is reduced in both head and following winds. Wave height is the major factor affecting ship performance. Wave action is responsible for ship motions, which reduce propeller thrust and cause increased drag from steering corrections. The relationship of ship speed to wave direction and height is similar to that of wind. Head seas reduce ship speed, while following seas increase ship speed slightly to a certain point, beyond which they retard it. In heavy seas, exact performance may be difficult to predict because of the adjustments of course and speed for ship handling and comfort. Although the effect of sea and swell is much greater than wind, it is difficult to separate the two in ship routing.

Fog, while not directly affecting ship performance, should be avoided as much as feasible, in order to maintain normal speed in safe conditions. Although the route may be longer by avoiding fog, transit time may be less due to not having to reduce speed in reduced visibility. In addition, crew fatigue due to increased watch keeping vigilance can be reduced. Ocean currents do not present a significant routing problem, but they can be a determining factor in route selection and diversion. The important consideration to be evaluated are the difference in distance between a great circle route and a route selected for optimum current, with the expected increase of speed of advance from the following current.

There are two main problems in the calculation of the optimal route. The first is the calculation of initial routes, especially in the case of the present of obstacles. The second is the definition of the route cost and route optimization. In this work, both problems are handled and applied to the operational system POLIS [1].

* Corresponding author: e-mail: dvlachos@ncmr.gr, Phone: +30 22910 76410, Fax: +30 22910 76323

2 Initial routes

The calculation of the initial routes is the most demanding computational aspect of the method. The basic criterion for the selection of the initial route is its length. In the simpler case, where no islands or obstacles are present, the initial route is the largest cycle which pass from the departure and arrival points. If an obstacle is present, then there are two possible ways to bypass it. In order to calculate the shortest route which bypass the obstacle, we make the following definitions:

Definition 2.1 A **tangent line** to an obstacle O from a point S outside the obstacle, is a line that has at least on common point with the boundary of the obstacle and no common points with the interior of the obstacle.

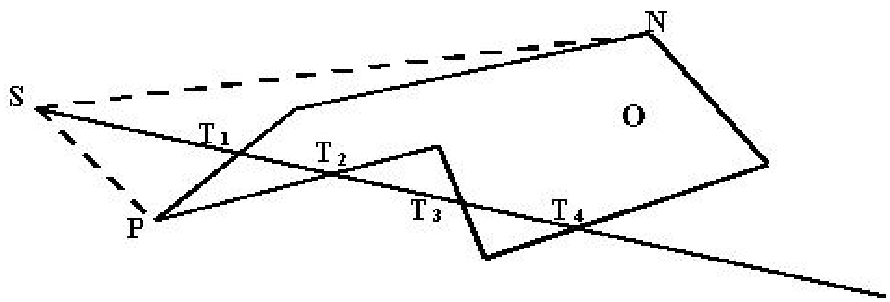

Fig. 1 Positive (SP) and negative (SN) tangent lines.

It is clear that from a point S there are two tangent lines to an obstacle O. Consider now that we have two points S and E and an obstacle O such that the line segment SE crosses the obstacle in points $T_1, T_2, ..., T_n$, where T_1 is the closest one to point S and T_n is the closest one to point E (figure 1). Moreover, consider the two tangent lines from S to O and two points on these lines P and N which belongs to the boundary of the obstacle. Starting from T_1 and moving on the boundary of the obstacle, we have to move in the counter clockwise direction (positive) to reach point P and in the clockwise direction (negative) to reach point N.

Definition 2.2 Referring to figure 1, the **positive** (**negative**) tangent line from S to O is the line SP (SN).

The shortest path from point S to point E can be represented by a list of points $\{S, I_1, I_2, ..., I_n, E\}$. The following proposition outlines the recurrent nature of our problem:

Proposition 2.3 *If a point B belongs to the shortest path $\{E, ..., S\}$ then the shortest path $\{B, ..., S\}$ belongs entirely to $\{E, ..., S\}$.*

The proof is obvious. The following theorem give us the necessary tool to calculate the shortest route which bypasses an obstacle.

Theorem 2.4 *Given two points S and E outside an obstacle O, the shortest path between S and E which does not intersects O is either the straight line SE either a union of tangent lines of the same sign.*

Proof. If the line SE does not intersects O then it is the shortest path. Otherwise, the point B which is defined by the positive tangent line from S to O belongs to the shortest path and so does the line segment SB. Thus, based on the above proposition, the shortest path $\{S, ..., E\}$ is the union of SB and the shortest path $\{B, ..., E\}$. The recurrent nature of the construction of the path proves the theorem. □

The above theorem gives us a systematic way to construct shortest paths containing either positive or negative tangent lines. When more than one obstacles are present, the total number of initial paths created is 2^n where n is the number of obstacles.

3 Route cost

By the term route cost we mean a scalar which is assigned in every possible route between two points. This scalar includes a weighted combination of the voyage time and the safety (or comfort) of the voyage.The total cost S is given by:

$$S = \alpha T + (1 - \alpha) C \tag{1}$$

where T is the total voyage time and C is a scalar characterizing the safety (comfort) of the voyage. The weight α can be tuned by the user depending on his demands. Note here that when $\alpha = 1$ then the only optimization parameter is the voyage time while when $\alpha = 0$ the only optimization parameter is the crew comfort. The scalar C is calculated as a line integral over the route by the following way (up to the linear approximation):

$$C = \int_A^B \left(\vec{w}^T \cdot Z_w + \vec{h}^T \cdot Z_h \cdot \right) \vec{dl} \tag{2}$$

where $\vec{w}$ is the wind vector, $\vec{h}$ is the wave height vector and Z_w and Z_h are tensors which characterize the ship response to wind and wave respectively.

4 Iterative route optimization

Consider now the initial route in figure 2. The route is braked into several line segments, the number of which depends on the resolution of the wind and wave forecasts. In this way, a route is represented as a list of way-points. Initially, the total cost of the route is calculated. Then, in every iteration step, every way-point of the route

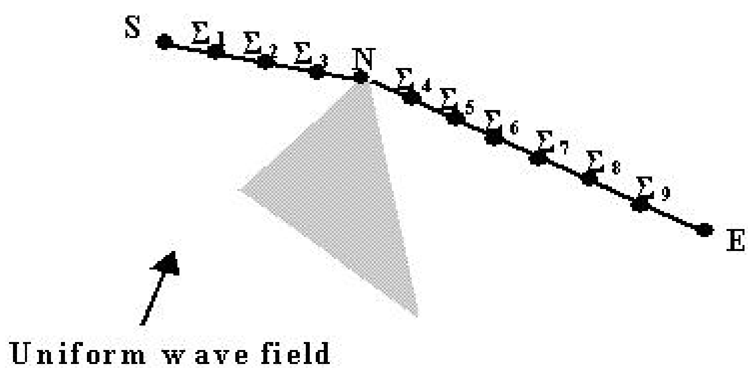

Fig. 2 Initial route between points S and E.

is moved by an elementary length perpendicular to the line which connects the departure and arrival points. This elementary length is specified by the resolution of the wind and wave forecasts. Every movement has a positive or negative contribution to the total cost. A movement is accepted even if it has a positive contribution to the total cost with a probability which depends on the temperature of the system. Initially, the temperature is high, but as the algorithm proceeds, the temperature is decreased to zero. At this point, only movements with negative contributions are accepted. This method is known as simulated annealing and is used to avoid local minima [2]. Notice here that there is no systematic way to decide if the calculated route is the optimal one. In most cases however, the voyage time is very critical and thus the optimal route is close to the shortest initial one. The only case that was observed in our experiments, where the iterative method was locked in a local minimum, was when the line between the departure and arrival points was very close to a small obstacle. In this case, the method could not overcome the obstacle and the algorithm terminated.

Figure 3 shows the calculated optimal route for a uniform wave field with w=0.5. Notice that the way-points are moved in order to maximize the product $\vec{v}^T \cdot \vec{w}$, where $\vec{v}$ is the velocity of the ship. This is very reasonable, since the higher moments around the ship's axis are developed when the wave is perpendicular to the ship's direction.

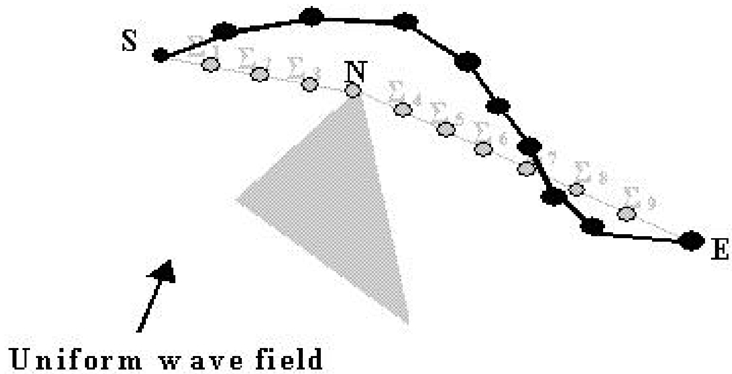

Fig. 3 Optimal route calculated for a uniform wave field with w=0.5.

5 Route optimization via simulated annealing

Another way to calculate the optimal route is the following: Assume that the ship is moving in the xy-plane. We can increase the dimensionality of the problem by considering the time as an extra dimension. Figure 4 shows this extension. We can create now a xyt-cube and try to solve the problem in its interior. First, we divide the cube in smaller cells with edges δx, δy and δt.

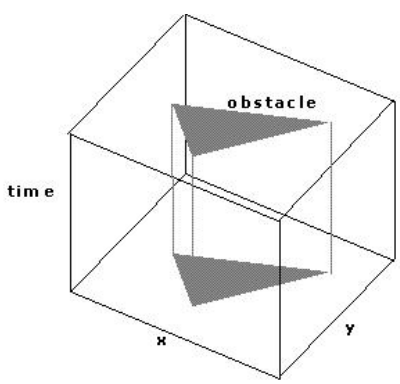

Fig. 4 Introducing time as an extra dimension to the optimization problem.

The problem now of the calculation of the optimal route can be formulated as the calculation of the optimal placement of N-balls in the cells which minimizes the cost function with the following constrains:

1. In each plane $t = const$ there is at most one ball. This means that the ship cannot be in two different places simultaneously.
2. In each line $x, y = const$ there is at most one ball. This means that the ship cannot pass twice from the same point.
3. The horizontal distance between balls in successive planes is exactly 1.
4. No empty planes $t = const$ exist between filled ones.

Suppose now that f_{xyt} has the value 1 if a ball is placed in the xyt-cell and 0 otherwise. The total cost can be written:

$$S = f_{xyt} \cdot H^{xyt}_{x'y't'} \cdot f^{x'y't'} \tag{3}$$

where $f_{xyt} = f^{xyt}$. The constrains now can be expressed as follow:

$$C_1 = \left(\sum_{xy} f_{xyt}\right) \cdot \left(1 - \sum_{xy} f_{xyt}\right) = 0 \tag{4}$$

$$C_2 = \left(\sum_{t} f_{xyt}\right) \cdot \left(1 - \sum_{t} f_{xyt}\right) = 0 \tag{5}$$

$$C_3 = f_{xyt} \cdot f_{x'y',t+1} \cdot (x - x') \cdot (y - y') = 0 \tag{6}$$

$$C_4 = \sum_{xy,x'y'} f_{xyt} \cdot f_{x'y',t+2} - \sum_{x''y''} f_{x''y'',t+1} = 0 \tag{7}$$

The cost function now can be rewritten as:

$$S' = S + \lambda_1 C_1 + \lambda_2 C_2 + \lambda_3 C_3 + \lambda_4 C_4 \tag{8}$$

Simulated annealing can now be applied, by letting the free variables $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ extend to infinity. Figure 5 shows the comparison of the iterative optimization (solid line) and the simulated annealing (dashed line) for different values of the user defined weight α. It is clear from this figure that simulated annealing gives better routes as α is increased but needs 4 to 5 times more computational time than the iterative optimization.

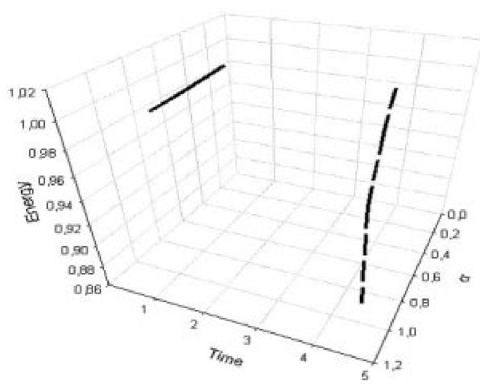

Fig. 5 Comparison of the iterative optimization (solid line) and the simulated annealing (dashed line) for different values of the user defined weight α.

6 Conclusions

Optimal ship routing is subject to environmental factors and good forecasting models must be used to produce satisfying routes. Basic results from computational geometry can be used to produce initial routes. Iterative optimization produces almost optimal routes in reasonable times. The main disadvantage of iterative optimization is that there are cases that a continuous transformation of an initial route cannot lead to the optimal one. On the other hand, by increasing the dimensionality of the problem by introducing time as an extra dimension, simulated annealing can produce better routes. The main disadvantage of simulated annealing is that it is for to five times mor time consuming than the iterative optimization.

Acknowledgements This work has been financial supported by the General Secretariat of Research and Development of the Greek Ministry of Development.

References

[1] D.S. Vlachos, Elsevier Oceanographic Series **69**, 649 (2003).
[2] A. Cichocki, R. Unbehauen, Neural Networks for Optimization and Signal Proccessing, John Wiley and Sons (1993).