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# Simulated annealing for optimal ship routing

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ABSTRACT

In this article we present a simulated annealing based algorithm for the determination of optimal ship routes through the minimization of a cost function defined as a weighted sum of the time of voyage and the voyage comfort (safety is taken into account too). This cost function is dependent on the wind speed and its direction as well as on the wave height and its direction. The constructed algorithm at the beginning discretizes an initial route and then optimizes it by considering small deviations, which are accepted or rejected by utilizing the simulated annealing technique. Using calculus of variations, we prove a key theorem which tremendously accelerates the convergence of the proposed algorithm. For an illustration of the advantages of the constructed method, both computational and real experiments have been carried out which are presented and discussed.



## 1. Introduction

Optimization of ship routing is closely related to both ship characteristics and environmental factors. Ship and cargo characteristics have a significant influence on the application of ship routing [1–3]. Ship size, speed capability and type of cargo are important considerations in the route selection process prior to sailing and the surveillance procedure while underway [5]. The characteristics of a ship identify its vulnerability to adverse conditions and its ability to avoid them [4,5,7].

As it is known, environmental factors of importance to ship routing are those elements of the atmosphere and ocean that may produce a change in the status of a ship transit [1,5]. The role of routing is to allocate the available resources (e.g. POSEIDON system, Ref. [6]) so as certain requirements are perfectly fulfilled. In ship routing studies, as environmental factors are considered the wind, sea waves, fog and ocean currents. While all the environmental factors are important for route selection and surveillance, a route is usually considered optimum if the effects of wind and waves are optimized [7,8].

The effect of wind speed on the ship performance is difficult to be determined. In light winds (less than 20 knots) ships lose speed in headwinds and gain speed slightly in following winds. For higher winds, ship speed is reduced in both head and following winds. Wave height is the major factor affecting ship performance. In general, the wave action is responsible for ship motions, which reduce propeller thrust and cause increased drag from steering corrections. The relationship of ship speed to wave direction and height is similar to that of wind. Head sea waves reduce the ship-speed, while following sea-waves increase ship-speed slightly up to a certain point, beyond which they retard it. In the case of strong sea waves, exact performance may be difficult to predict [1,7].

Concerning fog, while this is not directly affecting ship performance, it should be avoided as much as feasible, in order to maintain normal speed in safe conditions. Even though the ship route may become longer in order to avoid fog, transit time may be less, due to not having to reduce speed as it must be done in reduced visibility [4,7].

Ocean currents, generally, do not cause significant routing problems, but they can be a determining factor in route selection and diversion. The important considerations to be evaluated are the benefits of the optimal route, without taking into account ocean currents and a route selected for optimum current with the expected increase of speed from the following ocean current. More details about the effect of environmental factors to ship routing can be found in Refs. [1,5,6].

In this paper we concentrate on the problem of optimal ship routing taking into account only the wave height and its direction. For the optimization procedure we construct an appropriate simulated annealing based algorithm [9,10]. As it is known, a simulated annealing method is an extension of a Monte Carlo method developed by Metropolis et al. [11], to determine the equilibrium states of a collection of atoms at any given temperature *T*. Soon after the simulated annealing was first proposed in Ref. [10], intensive research effort has been devoted on its properties and applications [2,12,13].

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This technique attracted also significant attention as suitable for optimization problems of large scale. It can give solution when a desired global extremum is hidden among many, poorer, local extrema [14,15]. Even though other practical methods have also been developed for such purposes, surprisingly, the implementation of the algorithm in simulated annealing methods is relatively simple. The basic tool of such a method operates in analogy with thermodynamic processes and specifically with the way liquids freeze and crystallize, or metals cool and anneal when they are cooled slowly and thermal mobility is lost. For slowly cooled systems, nature is able to find the minimum energy state [10,11].

In the present work we exploit this property of physical systems to find the optimum route of a ship trip performed under the action of environmental factors and specifically under wind. In addition, we take advantage of the formulation of the discrete variational mechanics for the discretization of the initial ship route as it is described below.

The rest of the article is organized as follows. At first, in Section 2, the formulation of the problem is shortly outlined and the main features of the method based on discrete variational principles are briefly described. In Section 3 a key theorem, which provides an efficient way of searching for the optimal solution in ship routing, is proved. In Section 4, the operating steps of the simulated annealing algorithm constructed in this work are explained and analyzed. Both simulated and real experimental tests of the method are performed and the results are presented and discussed in Section 5. Finally, in Section 6 the main conclusions extracted in this work are summarized.

## 2. Brief description of the formalism

Let us assume that an initial route of a ship (see Fig. 1) is represented by a smooth curve $\vec{r}(s)$ where the parameter $s$ is the arc length measured from some fixed point $A$ (initial point of the ship route). Then, the tangent vector of the curve of the ship's route in the point of question is defined as (see e.g. Ref. [16])

$$\vec{t} = \frac{\dot{\vec{r}}}{|\dot{\vec{r}}|} = \frac{d\vec{r}}{ds} \tag{1}$$

where $\dot{\vec{r}}$ is the ship's velocity (see also [22,23]).

We also assume that the moving ship is subjected to the influence of the wave represented by (height and direction) the vector $\vec{w}$ and the wind represented by (speed and direction) the vector $\vec{v}$.

Under the above circumstances, we define a route cost function (a scalar quantity) assigned at every possible route between two points which includes a weighted combination of the voyage

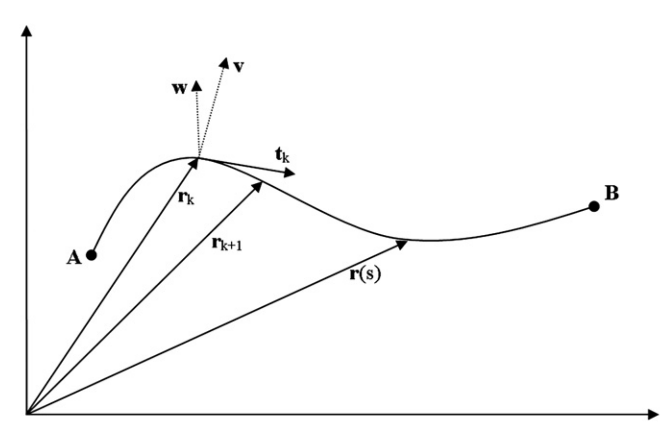


**Fig. 1.** A route from point $A$ to point $B$. $\vec{r}(s)$ is the parametrization of the route, and $w$, $v$ are the wave and wind vectors, respectively.

time and the safety (or comfort) of the voyage. The total route cost function $S$ reads

$$S = aT + (1-\alpha)C \tag{2}$$

where $T$ is the total voyage time and $C$ is a scalar characterizing the safety (comfort) of the voyage. The weight parameter $\alpha$ can be tuned by the user depending on the specific demands of the problem in question. Note that, when $\alpha = 1$, then the only optimization quantity is the voyage time, while when $\alpha = 0$, the only function to be optimized is the crew comfort $C$.

The scalar $C$ is calculated as a line integral over the route in the following way (up to the linear approximation) [6,7]:

$$C \equiv \int_A^B c\,ds = \int_A^B (\vec{v}^T Z_v + \vec{w}^T Z_w) \cdot \vec{t}\,ds \tag{3}$$

where $Z_v$ and $Z_w$ are tensors which characterize the ship response to wind (with wind vector $\vec{v}$) and wave (with wave vector $\vec{w}$), respectively. The calculation of the total voyage time $T$, is a bit more complicated, since both wind and waves can alter the speed of the ship (see Section 3). In general, for the magnitude of the ship speed we can write

$$|r(\dot{\vec{r}})| = u + F(\vec{v}, \vec{w}, \vec{t}) \tag{4}$$

where $u$ is the speed of the ship in zero wind and $F$ is a function that depends on ship characteristics, wind, wave and direction of the ship movement. For simplicity in the present work, we assume that $F = 0$.

### 2.1. The discrete variational principles

Recently, progress has been made in the development of variational discretization in mechanical problems, both in the fundamental theory and in the applications to challenging problems of optimization [17–20]. For such purposes, a method of variational integrators, based on the discretization of Hamilton's principle has been developed which underlines essentially the entire mechanics, from particle mechanics to continuum mechanics [17,18]. Also, the discretization of Lagrange–D'Alembert principle has been formulated and used, especially in cases where dissipation or external forces are present [18,19].

Preserving the basic variational structure in constructing an algorithm, it retains the structure properties of mechanics (such as conservation laws) at the algorithmic level. This avoids many of the problems appearing in some existing integrators, such as spurious dissipation, for which standard techniques may need very expensive runs in order to eliminate it [18].

With an appropriate development of the connection between mechanics and geometry through the discretization, one is able to use the methodology described in Ref. [20] for geometry, Refs. [17–19].

In the first step of the present work, we construct the comfort function $C$ which is subsequently treated with minimization techniques as in basic problems of the calculus of variation. This problem resembles to that of Hamilton's principle where a Lagrangian $L(q,\dot{q})$ is initially defined (for the specific mechanical system under consideration) to make the action integral stationary which leads to the Euler–Lagrange equations.

In the case of a ship routing problem, the role of the action integral plays the cost function $S$ of Eq. (2) or the comfort function $C$ of Eq. (3), for which we demand

$$\delta C = \delta \int_A^B [c(s)]\,ds = 0, \tag{5}$$

for all variations related to the route arc $s$ ($A$ and $B$ are the fixed end points of the route). One can mimic the procedure and exploit the advantages of the discrete variational mechanics in ship routing problems as demonstrated below.

## 3. Discretization of the initial ship's route

We start by considering the discrete case, where the ship route is approximated by a polygonal line with edges $\vec{r}_k$, $k=0,1,\ldots M$ (see Fig. 1). The length of each line segment is

$$\delta_k = |\vec{r}_k - \vec{r}_{k+1}|, \quad k=0,1,\ldots,M-1, \tag{6}$$

and the tangent vector to the route at each edge is

$$\vec{t}_k = \frac{\vec{r}_k - \vec{r}_{k+1}}{\delta_k}, \quad k=0,1,\ldots,M-1. \tag{7}$$

Then, the required ingredients for the cost function of Eq. (2) which corresponds to the route can be obtained by

$$T = \sum_{k=0}^{M-1} \frac{\delta_k}{u},$$

$$C = \sum_{k=0}^{M-1} \vec{z}_k^T \cdot \vec{t}_k \delta_k. \tag{8}$$

In the later equation

$$\vec{z}_k^T = \vec{v}^T Z_v + \vec{w}^T Z_w \tag{9}$$

is calculated at the point $\vec{r}_k$ and at time

$$T_k = \sum_{j=0}^{k-1} \frac{\delta_j}{u}. \tag{10}$$

Afterwards, the cost function of Eq. (2) reads

$$S = a \sum_{k=0}^{M-1} \frac{\delta_k}{u} + (1-\alpha) \sum_{k=0}^{M-1} \vec{z}_k^T \cdot \vec{t}_k \delta_k. \tag{11}$$

Consider now a small change in $\vec{r}_k$ of the form $\varepsilon \vec{x}_\phi$, where $\varepsilon$ is a small positive number and $\vec{x}_\phi$ is the unit vector in the direction which forms an angle $\phi$ with the horizontal axis. The new route will have cost $S'$ for which we have

$$\begin{aligned}\Delta S &= S' - S \\ &= a\varepsilon(\vec{z}_k^T - \vec{z}_{k-1}^T) \cdot \vec{x}_\phi + (1-a)\frac{\varepsilon}{u}(\vec{t}_k^T - \vec{t}_{k-1}^T) \cdot \vec{x}_\phi\end{aligned}$$

or

$$\Delta S = \varepsilon \left[ a(\vec{z}_k^T - \vec{z}_{k-1}^T) + (1-a)\frac{1}{u}(\vec{t}_k^T - \vec{t}_{k-1}^T) \right] \cdot \vec{x}_\phi \tag{12}$$

To obtain the later equation we have assumed that

$$|\vec{r} + \vec{\lambda}| \approx |\vec{r}| + \frac{\vec{r} \cdot \vec{\lambda}}{|\vec{r}|}, \quad |\vec{\lambda}| \ll |\vec{r}|. \tag{13}$$

In our simulated annealing method the expression of $\Delta S$ of Eq. (12) enters the Maxwell Boltzmann probability distribution in order to define optimized routes, as it is described below and in Section 4.

### 3.1. Searching using simulated annealing

We consider that the initial route illustrated in Fig. 1 is divided into several line segments, the number of which depends on the wind and wave forecasts. In this way, any ship route is represented by a set of way-points. The total cost of the initial route is readily calculated. Then, at any iteration step, every way-point of the route is moved by an elementary length perpendicular to the line which connects the departure and the arrival points (end points of the segment). This elementary length is specified by the resolution of the wind and the wave forecasts.

Every displacement has a positive or negative contribution to the total cost and it is accepted even if it has a positive contribution to the total cost with a probability, however, which depends on the temperature-parameter of the algorithm which pays the same role as the temperature of a physical system.

In simulated annealing methods the probability distribution is described by the so-called Boltzmann probability distribution,

$$p(E) \equiv Prob(E) \approx \exp(-E/kT). \tag{14}$$

(the quantity $k$, Boltzmann's constant, is a constant of nature that relates temperature to energy). The physical meaning of the latter equation is that a system in thermal equilibrium at temperature $T$ has its energy states probabilistically distributed according to their energy $E$. Even at low temperature, there is a chance, albeit very small, for a physical system to occupy a high energy state. Correspondingly, there is a chance for the system to get out of a local energy minimum in favor of finding a better, more global energy minimum. In other words, the system sometimes goes uphill as well as downhill, but the lower the temperature, the less likelihood for any significant uphill transition.

Initially, the temperature-parameter of the algorithm is high, but as the algorithm proceeds, the temperature is decreased to zero. At this point, only movements with negative contributions are accepted. This method, known as simulated annealing, is used to prevent the algorithm from being trapped in local minima through the above mechanism it proceeds further to find the global minimum (or minima).

Notice here that, there is no systematic way to decide if the calculated route is the optimal one. In most cases, however, the voyage time is very critical and, thus, the optimal route is close to the shortest initial one. The only such case, in which we observed in our experiments (the iterative procedure was locked in a local minimum), appeared in the case when the line between the departure and arrival points was very close to a small obstacle (see Fig. 2). In the majority of such instances, the method could not bypass the obstacle and the algorithm terminates. The reason is that only continuous transformations of the route are permitted and performed at any step of the algorithm. To overcome such difficulties, we consider several initial routes, as it will be explained in the next section.

### 3.2. Acceleration of the convergence in the searching mechanism

In order to accelerate the convergence of the searching mechanism, it would be useful to know in advance, to which directions the cost function is more sensitive to the transformations of the initial route. Fortunately, one can prove that, in minimizing the cost function for any segment of the ship's route, the resulting movement is parallel or anti-parallel to the direction of the wave propagation.

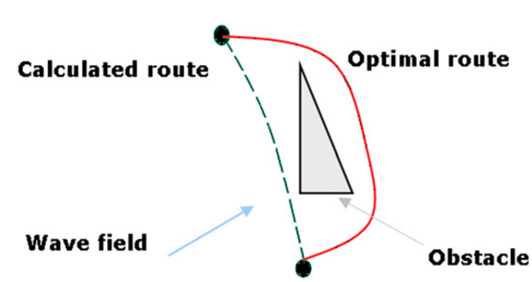


**Fig. 2.** The solution can be locked in a local minimum if the optimal route cannot be generated by continuous transformations of the initial route.

The proof, by considering only the contribution of the wave field in the cost function of Eq. (8), proceeds as follows.

Under the above conditions, the cost function of Eq. (8) is given by

$$C=\int_A^B(\vec{w}^T Z_w)\cdot\vec{t}\,ds \tag{15}$$

and we assume a change of the route close to the point $\vec{r}(s_0)$ which results in a new tangent vector of the form

$$\vec{t}'=\vec{t}+\varepsilon\delta(s-s_0)\vec{x}_\phi \tag{16}$$

($\delta$ is the Dirac function or $\delta$-function) where $\varepsilon$, $\vec{x}_\phi$ have been defined in Section 3.1. The variation in the cost function which corresponds to

$$\delta\vec{t}=\varepsilon\delta(s-s_0)\vec{x}_\phi \tag{17}$$

is written as

$$\delta C=\int_A^B(\vec{w}^T Z_w)\varepsilon\delta(s-s_0)\cdot\vec{x}_\phi\,ds \tag{18}$$

or

$$\delta C=\varepsilon\vec{z}_{|s=s_0}\cdot\vec{x}_\phi \tag{19}$$

where $\vec{z}_{|s=s_0}=\{\vec{w}^T Z_w\}_{|s=s_0}$, i.e. the vector $\vec{w}^T Z_w$ calculated at the point of the route where $s=s_0$.

It is obvious from Eq. (19) that the maximum change of the cost function will occur if the angle between the vector $\vec{z}_{|s=s_0}$ and $\vec{x}_\phi$ is 0 or $\pi$. Hence, the acceleration of the convergence is achieved by selecting variations of the initial route only in the above directions. In our experiments, the direction of the vector $\vec{z}_{|s=s_0}$ is always the same with the direction of the wave propagation, thus, the accepted variations of the initial route are those which are parallel or anti-parallel to the direction of the wave field.

## 4. Description of the new algorithm

### 4.1. Selection of the initial route and its representation with way-points

Before starting the optimization procedure with the algorithm, we have to provide one (or several) initial route. In the case when there are no obstacles between the starting and ending points, the initial route is simply the line joining these two points. Furthermore, if there is one obstacle between the starting and ending points, then there are two possible initial routes, each of which bypasses the obstacle from a different side. It is a good practice to start with shortest initial routes (one route for every possible bypass), thus having already optimized the voyage time. Obviously, in this case, if a point $B$ belongs to the shortest path between points $A$ and $C$, then the shortest path between points $B$ and $C$ is part of the shortest path between points $A$ and $C$ [24,23].

It is proved in Ref. [7] that the shortest path bypassing an obstacle is tangent to the convex hull of the obstacle. Thus, the problem of calculating the shortest path is reduced to the calculation of the shortest path from the contact point of this tangent to the ending point.

If there is more than one obstacle, recursively, we get that the number of initial paths is $2^n$, where $n$ is the number of obstacles between the starting and ending points. The algorithm must check all the initial routes to decide for the optimal solution even if for these checks the computational time has to be significantly increased.

As soon as the initial route is calculated, one has to decide for the number of way-points which will be used to represent the route. From a first view, one can think that this number plays the same role with that of the degree of a polynomial employed to fit a given smooth curve. In the latter case, increasing the degree of the polynomial, results to a better fitting, but this is not exactly the case here for the following reasons.

(1) The first reason is that, increasing the number of way-points, the computational time is increased (in a real ship trip the actions that have to be taken by the ship's crew are increased too).
(2) The second reason concerns with the period $T_d$ of the forecast data. If $u$ is the speed of the ship, then nothing is known about the drift of the environmental data in the time interval $T_d$ and, thus, in the space interval $L_d=uT_d$.
(3) The last reason is related to the details of the model used to forecast the environmental data, and more precisely the mesh that is used. If $L_d$ is the distance of the mesh points, then the ship for the time interval $L_d/u$ is moved in constant environmental parameters.

Experiments performed with various numbers of way-points have shown that an optimal selection for the distance of the way-points should be close to $L_d$. It is, then, a good practice to continuously adjust the number of way-points. If the distance between two successive way-points is greater than a chosen $D_M$, another way-point is added between them. On the other hand, if two successive way-points are closer than a special distance $D_m$, the two way-points are merged in one. In the experiments carried out and presented in this paper (see Section 5 below) we have chosen

$$D_M=2\cdot L_d,\quad D_m=L_d/2 \tag{20}$$

### 4.2. Stages of the algorithm

The calculation of the optimal route is afterwards performed through the following steps:

1. Consider the set $\{\vec{r}_k, k=0,(1),M\}$ of $M+1$ points representing the initial route.
2. Choose a random integer $k$ between 1 and $M-1$, i.e. including all points of the set except the end points corresponding to $k=0$ and $k=M$. Generate a random real $\phi$ such that $0\le\phi\le 2\pi$.
3. Calculate the difference of the cost function, i.e. Eq. (12), between the initial route and the route resulting by shifting $\vec{r}_k$ by $\varepsilon\vec{x}_\phi$, where $\varepsilon$ is a small real number and $\vec{x}_\phi$ is a unit vector in the generated random direction.
4. Generate a random number $p$ between 0 and 1 and accept the variation of the route if $p<e^{-\Delta S/T}$, where $T$ stands for the temperature-parameter. If the variation is not accepted, goto step 6.
5. Check $\vec{r}_k$ with the adjacent points, which means that, if two points are very close, merge them or, if they are too far, add a new point between them according to Eq. (20).
6. Decrease progressively the temperature-parameter $T$ of the simulated system in analogy with the cooling of the physical system (simulated annealing). Check if the algorithm has to terminate, otherwise goto step 2.

The symbols used above have been explained previously, for further details the reader is referred to Ref. [21–23].

## 5. Numerical results

In this section, we apply the new algorithm, first, to carry out two computational experiments, one in which we look for the

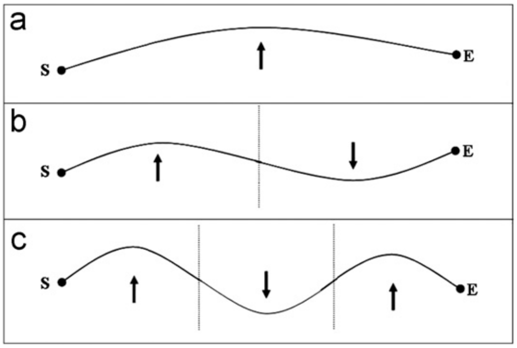


**Fig. 3.** Optimal route calculated as follows: (a) in a wave field with constant magnitude and direction, (b) in a wave field with constant magnitude but with direction which is inverted once during the voyage, and (c) same as in (b) but the direction is inverted twice.

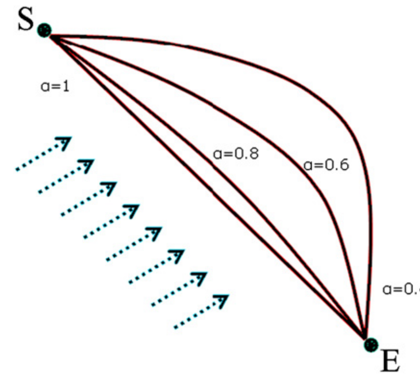


**Fig. 4.** The role of the weight parameter $\alpha$ in the cost function $S$. If $\alpha=1$, the calculated route is simply the line joining the starting and ending points. By decreasing the value of $\alpha$, the contribution of the voyage time into the total cost is decreased and the calculated route becomes a curved path.

optimal route under constant (or piecewise constant) wave field, and another (simulated) experiment in which the weight parameter $\alpha$ is varied. In the second stage of our applications, by using our method, we search for the optimal route in a real trip of a ship from Thessaloniki to Agios Nikolaos which is performed under piecewise constant wave field (for more details see [25]).

### 5.1. Simulated experiment under constant wave field

In the first simulated experiment, the optimal route under the action of a uniform wave field, see Fig. 3(a), is calculated.

In this case, see Fig. 3(a), the direction of the uniform wave field, $\vec{w}$, was assumed upwards and it does not change during the voyage. Notice that the way-points are moved upwards in order to maximize the product $|\vec{u}\cdot\vec{w}|$, where $\vec{u}$ is the velocity of the ship (in the real ship trip this is very reasonable, since the higher moments around the ship's axis are developed when the wave is perpendicular to the ship's direction).

In the second simulated experiment the optimal route for a piecewise uniform wave field, cases of Figs. 3(b) and (c), is calculated. We have assumed that the wave field is inverting direction once and twice, respectively. The observed behavior, demonstrated in Figs. 3(b) and (c), for each part of the route is similar to that of Fig. 3(a).

### 5.2. The effect of the weight parameter $\alpha$

In this computational experiment we studied the effect of the weight parameter $\alpha$, which enters the definition of the cost function $S$ of Eqs. (2) and (11). The wave field is assumed constant with direction perpendicular to the line joining the starting ($S$) and ending ($E$) points of the voyage, as indicated in Fig. 4.

When the parameter $\alpha$ takes its largest value ($\alpha=1$), the algorithm is not taking into account the comfort of the voyage. Since voyage time is the only parameter to optimize, the calculated value is simply the line joining the starting and ending points.

By decreasing the value of the parameter $\alpha$, the comfort of the voyage starts to act cumulatively to the total cost function, thus, forcing the algorithm to turn the ship parallel or anti-parallel to the direction of the wave field. This tendency is compensated by the time of the voyage which is increased. By decreasing more the value of the parameter $\alpha$, the contribution of the voyage time into the total cost is decreased, leading to longer routes which are aligned with the direction of the wave field (see curved paths of Fig. 4).

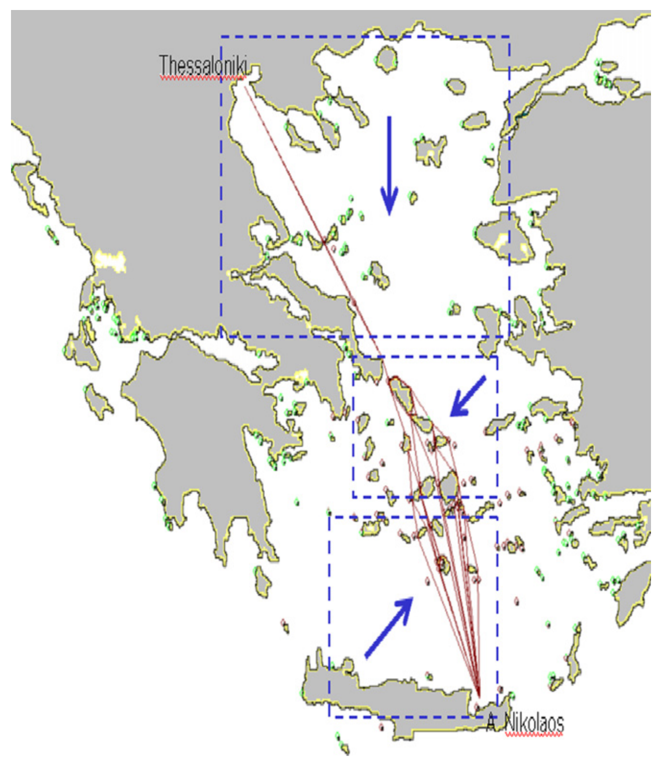


**Fig. 5.** Initial routes calculated from the port of Thessaloniki to the port of A. Nikolaos. The dominant wave fields are shown across the routes with an arrow showing the direction and magnitude of the wave field.

### 5.3. Optimal route of real ship trip under piecewise uniform wind field

In the last experiment, we test our new method on the calculation of the optimal route of a ship performing the trip from the port of Thessaloniki (40.5197N, 22.9709E) to the port of Agios Nikolaos (35.1508N, 25.7227E). The forecast data have been obtained from the *POSEIDON* system which uses floating buoys for real time measurements and mathematical models to predict the wave characteristics for the next 48 h [8]. In this experiment we have to take into account several initial routes, since there are many obstacles (islands) each of which doubles the number of total initial routes [7].

The initial routes are plotted in Fig. 5 with the dominated wave direction across the routes of this real physical problem.

**Fig. 6.** Optimal route calculated from the port of Thessaloniki to A. Nikolaos for the case shown in Fig. 5 with the proposed algorithm (solid line) and with the application of genetic algorithms (dashed line).

In this figure there are three main regions, the first one is dominated by waves coming from the North (this is the region close to Thessaloniki), the second is dominated by waves coming from North-East (the middle region in the Aegean sea) and the third is dominated by waves coming from South-West (this is the region close to Agios Nikolaos).

Fig. 6 shows two optimal routes: the obtained with our new algorithm (solid line) and the calculated by applying genetic algorithms (dashed line). The value of the weight parameter was $\alpha = 0.5$. The two routes agree except for a small part at the beginning of the voyage. This is mainly due to the property of the route calculated with the proposed algorithm to be continuously transformed from the initial route. Nevertheless, the difference in the total cost of the two routes is not significant.

An interesting trend of the optimal route calculated with our new algorithm is that the ship is trying to "*hide*" behind the obstacles (islands) where the magnitude of the wave height is significantly decreased.

## 6. Summary and conclusions

An effective operational algorithm for the calculations of optimal ship routes has been developed. The algorithm is based on the simulated annealing technique, a basic tool in searching for optimal solutions. As it was found, the searching process is accelerated by considering advantageous choices of variations of the initial route, i.e. those which are parallel or anti-parallel to the direction of the wave field.

Wind field tests, simulated and real, have been performed to demonstrate the efficiency of the proposed algorithm. The evaluated optimal routes have been compared with those obtained by exhaustive and time consuming genetic algorithms. Even though, in general, the differences are not significant, with the use of the present algorithm some new features of the optimal route are revealed (e.g. in the presence of the wind field the optimal route "hides" behind the obstacles).

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