Journal of Statistical Mechanics:
Theory and Experiment

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To cite this article: D Xenides *et al J. Stat. Mech.* (2008) P07017

Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

Synchronization in complex systems following a decision based queuing process: rhythmic applause as a test case

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Received 5 March 2008
Accepted 27 June 2008
Published 18 July 2008

Online at stacks.iop.org/JSTAT/2008/P07017
doi:10.1088/1742-5468/2008/07/P07017

Abstract. Living communities can be considered as complex systems, and are thus fertile grounds for studies related to statistics and dynamics. In this study we revisit the case of rhythmic applause by utilizing the model proposed by Vázquez *et al* (2006 *Phys. Rev.* E **73** 036127) augmented with two opposing *driving forces*, namely the desires for *individuality* and *companionship*. To that end, after performing computer simulations with a large number of oscillators we propose an explanation on the following open questions: (a) Why does synchronization occur suddenly? (b) Why is synchronization observed when the clapping period (T_c) is $1.5 \times T_s < T_c < 2.0 \times T_s$ (T_s is the mean self-period for the spectators) and lost after a time? Moreover, on the basis of the model, a weak preferential attachment principle is proposed which can produce complex networks obeying a power law in the distribution of the number of edges per node with exponent greater than 3.

Keywords: population dynamics (theory), critical phenomena of socio-economic systems, interacting agent models, stochastic processes

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Contents

1. Introduction

Periodic phenomena are of great abundance in Nature; what makes them attractive to physicists is the fact that they can serve as prototype complex networks—thus the importance of understanding their statistics and dynamics. To give an idea of the universality of the phenomenon we mention, as examples, the *Pteroptyx malaccae* fireflies [1], secretory cells [2], synchronously firing neurons [3], and rhythmic applause [4]. In particular, the study of mechanisms that lead to self-organization of biological systems through synchronization is of major importance, since it provides enough information for understanding the dynamics of the interactions among the members of the system.

To address the problem of synchronization we used the coupled oscillators approach. It is believed that Huygens' ideas about the synchronous movement of wall-hung pendulums initiated studies aimed at finding the explanation of this phenomenon. It is noted that coupling of identical oscillators is trivial since it can be achieved by a phase-minimization procedure. This is not the case in biological systems since the oscillators can be considered anything but equivalent. In such diverse systems, depending on the strength and type of interaction as well as the dispersion of the self-periods, synchronization might or might not appear. To illustrate the problem we refer to the synchronous clapping of an audience after a spectacle. The audience comprises a large number of spectators, each characterized by their temperament, enthusiasm, acoustic behavior; thus each can be considered to applaud for their own unique self-period.

A first approach to rhythmic applause indicates that an integrate-and-fire-type model [5, 6] is the most promising for understanding the synchronization mechanism. However, this model does not take into account memory effects which are crucial for the synchronization to occur; thus the continuous phase coupling in the Kuramoto model [7] has been used to understand the underlying dynamics [8]. On the other hand, a recent study brought forth strong evidence that human dynamics of many social, technological and economic phenomena can be modeled by a decision based queuing process [9] in which individuals execute a task from a list of pending tasks based on a given priority.

The purpose of the present paper is to show that application of the aforementioned decision based queuing process is capable of modeling the dynamics of rhythmic applause and especially: (i) why synchronization occurs suddenly, (ii) why synchronization is

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achieved in a period almost two times the average self-period of the spectators, and (iii) why synchronization appears and disappears several times during the applause.

2. Rhythmic applause and the Kuramoto model

The Kuramoto model has been used to model the physics of rhythmic applause. In this model we deal with N oscillators, each of them described by its ϕ_j phase. The oscillators have a $g(\omega)$ distribution of their ω_j self-frequency (where $g(\omega)$ is considered to be a normal distribution). Every rotator interacts with all others via phase-difference-minimizing terms obeying the following formula:

$$W_j^{\mathrm{int}} = \frac{K}{N}\sum_{i=1}^{N}\sin(\phi_i - \phi_j). \tag{1}$$

The N coupled differential equations describing the overdamped oscillator dynamics are

$$\frac{\mathrm{d}\phi_j}{\mathrm{d}t} = \omega_j + \frac{K}{N}\sum_{i=1}^{N}\sin(\phi_i - \phi_j). \tag{2}$$

Mathematically, the synchronization level will be characterized by an order parameter, q, defined at any time moment as

$$q = \left|\frac{1}{N}\sum_{j=1}^{N}\mathrm{e}^{\mathrm{i}\phi_j}\right|. \tag{3}$$

The maximal possible value $q = 1$ corresponds to total synchronization, the case $0 < q < 1$ to partial synchronization, while for $q = 0$ there is no synchronization at all in the system.

For the $N \to \infty$ thermodynamic limit of the equilibrium dynamics ($t \to \infty$, so initial transient effects are lost), Kuramoto and Nishikawa proved the existence of a K_{c} critical coupling. For a Gaussian distribution of the oscillators' natural frequencies, characterized by a dispersion D, they got

$$K_{\mathrm{c}} = \sqrt{\frac{2}{\pi^3}}D. \tag{4}$$

For $K \le K_{\mathrm{c}}$, the only possible solution gives $q = 0$ (no synchronization) while for $K > K_{\mathrm{c}}$ a stable solution with $q \neq 0$ appears. Thus, the main result is that for a population of globally coupled non-identical oscillators a partial synchronization of the phases is possible whenever the interaction among oscillators exceeds a critical value.

There are four major drawbacks of this model when applied to rhythmic applause:

(1) The uniform one-to-one coupling between all the oscillators. The spatial distribution of spectators prohibits this assumption.

(2) The coupling constant should be $K \ge K_{\mathrm{c}}$. Since (i) synchronization is achieved at a period $T_{\mathrm{c}} \approx 2 \times T_{\mathrm{s}}$ (with T_{s} the mean self-period of the spectators), and (ii) the doubling of the period leads to the half-doubling of the dispersion coefficient (D), the following inequality:

$$(1/2)\sqrt{2/\pi^3}D < K < \sqrt{2/\pi^3}D \tag{5}$$

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should hold. We note that D is strongly related to behavioral characteristics of the spectator (temperament, enthusiasm), and is thus unrelated to K which, however, is related to factors such as the distribution of the sound in the room, reflections of the sound etc. On the other hand, the 'generalized relaxation oscillators' approach is characterized, again, by the need for a global coupling among the oscillators that cannot be guaranteed in networks of living organisms.

(3) Usually at the beginning of the applause there is a long 'waiting' time without any synchronization, and with no increase in the order parameter. Partial synchronization evolves suddenly after that and achieves its maximal value in a short time. This should not be the case if we are in the $K > K_c$ limit. One would expect in this limit a continuous increase in the order parameter right from the beginning of the applause.

(4) Why is synchronization already achieved lost after a time, and why might it reappear again? Loss of synchronization should not happen in the $K > K_c$ limit.

3. The decision based queuing process

Recently, Vázquez *et al* [9] proposed a decision based queuing process (DBQP) able to reproduce the distribution of the possible action of humans in particular cases. In this model, all the possible actions form a list and the one performed is chosen by applying criteria (e.g., priority) related to the specific individual; that is, the emotional factors play an equally important role. In particular, most of the time humans are asked to decide on multiple tasks ranging from work to entertainment and from family responsibilities to everyday duties. To accomplish these tasks humans maintain a *to do* or *priority* list. Depending on the individual, this list could be written down, but most of the time it is held in the memory. It is worth noting the dynamical nature of this list, meaning that tasks are either executed or become irrelevant, and tasks are added continuously. Furthermore, tasks compete for the individual's time and attention. For these reasons, the management of the tasks could be seen as similar to a queuing process where the queue represents the *priority* list, the server is the human who executes them and maintains the list, and some selection protocol governs the order in which the tasks are executed.

Applying the above DBQP model in the case of rhythmic applause, the spectator has to chose among the following actions (tasks):

(1) to clap at a frequency to her/his satisfaction (this an expression of *individualism*), or

(2) to try to change her/his pace so as to follow the clapping frequency of the synchronized spectators (this an expression of *companionship*); it is obvious that this holds true only in the presence of a group of people.

These two tasks form the *priority* list mentioned in the DBQP model. The only difference is that tasks are not removed from the list after selection and execution. As far as the task selection protocol is concerned, we note that a dynamical process controls the priority level of each task. More specifically, we have the following.

Every time that a spectator claps, she/he makes a decision between (1) and (2), depending on the priority given to each of them. The priority is immediately related to the level of satisfaction reached, reflected in the desire for expressing her/his *individualism* or her/his *companionship*. We name the first I_{prio}. The fluctuation of this parameter with

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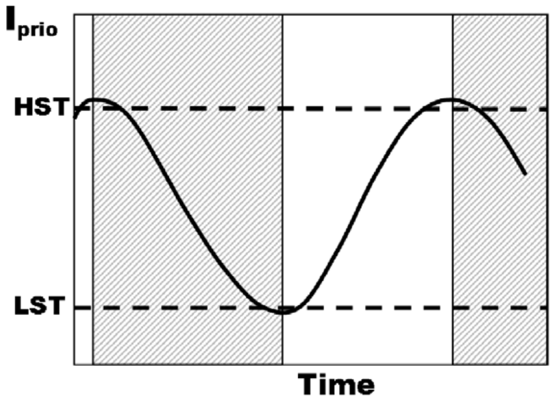

Figure 1. The priority I_{prio} of spectators for expressing their *individualism* as a function of time. When I_{prio} climbs over HST then the desire is considered saturated and I_{prio} starts to decay. The opposite behavior for I_{prio} is observed when it reaches LST, and thus the desire comes again into the foreground, causing I_{prio} to increase.

the time is ruled by the level of saturation: for saturation, I_{prio} reduces with time, whereas it increases otherwise. Two boundaries control this behavior, namely the high saturation threshold (HST) and the low saturation threshold (LST). When I_{prio} climbs over HST, then the spectator's desire to express her/his *individualism* is considered to saturate; thus the decay of I_{prio}. On the other hand, when I_{prio} falls under LST, then the desire to express the *individualism* comes again into the foreground; thus I_{prio} starts to increase. With these mechanisms we try to simulate the behavioral transitions of the spectators from a phase where they are expressing enthusiasm to another phase where they have either fulfilled this desire or something else has attracted their attention. This mechanism is qualitatively depicted in figure 1.

As regards the priority of the second task (the expression of *companionship*), we name this as C_{prio}. The level of C_{prio} is strongly related to the synchronization level observed in the audience. If the synchronization level is low, the spectator feels only an intense background noise which does not enable her/him to take any special action to change her/his applause rhythm. On the other hand, if the synchronization level is high and thus the background noise is low, then there is a significant probability that the spectator will try to follow the dominant sound. Though I_{prio} is artificially varied with time, it is C_{prio} that is fully characterized by the level of the background sound.

A model system of a square grid with $n \times n = n^2$ spectators is chosen and it is assumed that the sound produced in position (i, j) is heard weakened, proportional to the distance from the source, in neighboring regions. Thus, the intensity of the sound in position (i, j) at time t is

$$S(i,j,t) = \sum_{i'=1}^{n} \sum_{j'=1}^{n} \delta(i',j',t) K(i-i') K(j-j') \tag{6}$$

where K is the sound weakening factor (a five-point derivative kernel [10]), and $\delta(i', j', t')$ takes values 1 and 0 if at time t the spectator at position (i', j') claps or does not clap her/his hands, respectively. We assume that at time t the sound intensity in the

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Figure 2. Different moments during the *decision making* interval.

(i, j) position is $S(i, j, t)$ and $\delta(i, j, t) = 1$, meaning that at that time the spectator at position (i, j) claps her/his hands. It can be shown that $S(i, j, t)$ could give us information on the total number of synchronized clapping spectators. At time t' when $\delta(i, j, t') = 0$ but $S(i, j, t') > S(i, j, t)$ the sound heard at position (i, j) is stronger than that at time t when the (i, j) spectator also acted. There emerges the desire of expressing her/his *companionship*. The greater the $S(i, j, t')/S(i, j, t)$ ratio, the higher the priority given to this action, as explained above. Thus, $C_{\rm prio}$ at each site (i, j) is controlled by this ratio. On the other hand, $I_{\rm prio}$ is initialized randomly at each site (i, j) and then is increased or decreased according to the mechanism already explained before.

At any given moment t a random number P is generated: if $P < (I_{\rm prio}/(I_{\rm prio} + C_{\rm prio}))$ then the action (1) is chosen; otherwise the action (2) is chosen.

Let us see what happens when the action (2) is chosen. If t is the present time and t_a the time (before t) when $\delta(i, j, t_a) = 1$ and t_b the time (after t) when $\delta(i, j, t_b) = 1$ again, then $t_b = t_a + T_{i,j}$, with $T_{i,j}$ the self-period of the spectator (i, j) (see figure 2).

The following three cases might occur:

- If t is closer to t_a, the spectator increases her/his self-period hoping that she/he will be synchronized with the following clapping.
- If t is closer to t_b, the spectator lowers her/his self-period hoping that she/he will be synchronized with the following clapping.
- If t is closer to $(t_a + t_b)/2$, the clapping at that time will result in an increment of the background noise that is strongly related to the synchronization. In view of that, (i, j) wishes not to increase the background noise and thus suspends her/his clapping at time t_b (we note that this could happen only once).

4. Experimental results—discussion

A statistical ensemble resembling a grid of 625 (25×25) spectators was simulated. We begin the simulation with self-periods following a normal distribution with average value 50 and $\sigma = 10$. We assume that immediately after the clapping starts, spectators feel a great desire for expressing their enthusiasm or *individualism*; thus $I_{\rm prior}$ is high. For this reason spectators retain their self-periods of clapping and consequently there are no signs of synchronization. As time passes, $I_{\rm prior}$ gets lower, resulting in the forming of random, at the beginning, clusters, that give rise to the expression of *companionship*. Immediately after the forming of the first cluster an abrupt increase of the average period is observed as shown in figure 3. This is in agreement with results presented in [4, 8].

The critical point $t_{\rm c}$ of this abrupt change of period ($t_{\rm c} \approx 2000$) is characterized by a domination of the $C_{\rm prio}$ over $I_{\rm prio}$. After some time, $I_{\rm prio}$ starts to increase because it reaches the LST. Spectators are starting to express themselves again by clapping at

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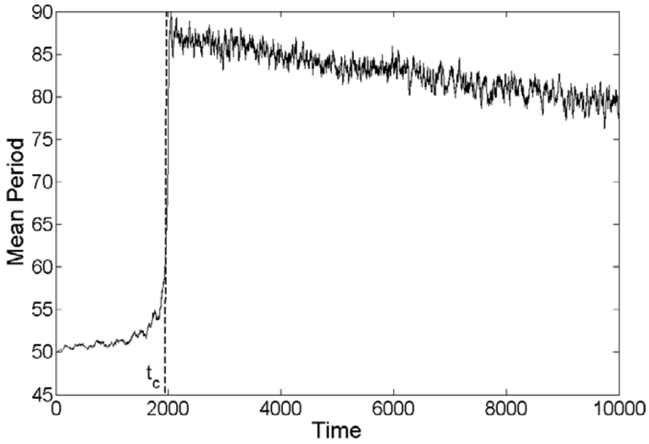

Figure 3. The average period for an audience of 625 spectators in a rectangular 25×25 grid. The self-periods follow a normal distribution with mean value 50 and $\sigma = 10$. The simulation parameters are: HST $= 0.9$, LST $= 0.05$, $\mathrm{d}I_{\mathrm{prio}}/\mathrm{d}t = \pm 0.001$.

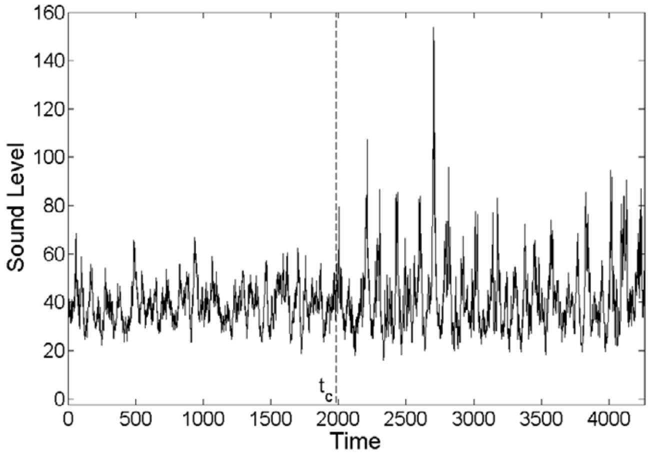

Figure 4. The sound level at position (12, 12) of the audience for the same simulation as in figure 3. Synchronization after the critical time t_{c} is indicated by the recorded spikes.

their eigenfrequency, resulting in the loss of synchronization. Later, I_{prio} starts to decay because it reaches the HST and synchronization might occur again. This mechanism might be repeated several times during the clapping period as is found experimentally [8].

Figure 4 shows the sound level at the position (12, 12) of the audience. At the critical time t_{c}, synchronization is visible as pronounced spikes in accordance with experimental findings [8]. The dominant period after time t_{c} can be evaluated by counting the spikes, resulting in a value of $\approx$100 which is in agreement with the findings of figure 3.

Moreover, it seems that the mean period of the spectators is a function of LST: the maximum mean period recorded in the simulation versus the LST is depicted in figure 5. The error bars are proportional to the background sound, which is a measure of synchronization as is explained in [4].

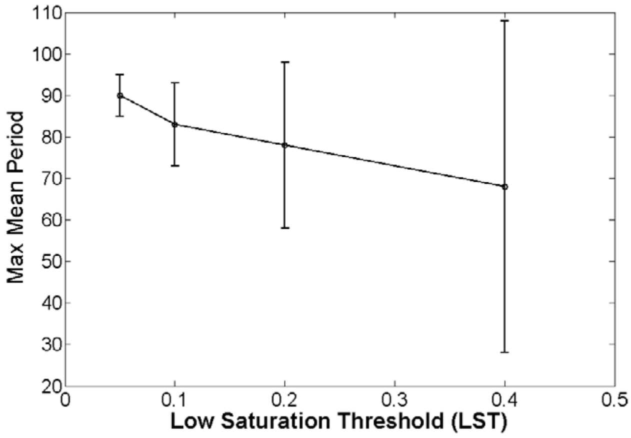

Figure 5. The maximum mean period recorded just after the critical time t_c, of the same simulation as in figure 3, versus the level of the low saturation threshold LST. The error bars are proportional to the background noise which is considered to be a measure of lack of synchronization.

It is interesting to follow the dynamics of the abrupt change of the mean period shown in figure 3. Initially, lack of synchronization is due to the high value of I_{prio} as was explained before. After some time (the value of I_{prio} has been decreased), random clusters of synchronized spectators may be formed producing locally a high sound level. Spectators near those clusters either slowly start to take action to synchronize with them or elect to suspend some of their claps. In the latter case, the background noise is decreased (since some claps are missing) causing C_{prio} to increase suddenly. This effect acts as an avalanche making the other spectators act faster to synchronize with the already formed clusters, since their sound will now be more evident. Hence, the aforementioned mechanism is expected to have a stronger effect on those spectators who clap at higher frequencies, since these spectators contribute more to background noise. For this reason, low clapping frequencies are sustained and the dominant period during synchronization is increased in comparison to the mean self-period of the spectators.

Furthermore, we note that the switching between synchronous and asynchronous clapping is fully controlled by the C_{prio} to I_{prio} ratio. In addition, C_{prio} values depend only on the background sound whereas I_{prio} takes various values as was mentioned above. Concerning the mean period of clapping during synchronization, the following observations might provide a possible explanation for the almost doubling of the mean period of the spectators. Let us examine the case where the dominant period near a spectator is exactly two times her/his own; then when the spectator decides to suspend her/his clapping this will be reproduced continuously, making it impossible for the spectator to become synchronized. On the other hand, if the dominant period is slightly less than the double, then the spectator after a few cycles will be synchronized. Moreover, the higher the mean period of synchronization the greater the number of spectators (who contribute to the background noise) who will act so as to synchronize. In contrast, when the mean period is more than two times, although the number of possible active spectators could be larger, at the same time the actions occurring in a period will cancel each other (if the spectator decides to increase her/his period next time, she/he will decide to decrease it). Thus, synchronization is not likely to occur.

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Our proposal was further tested for validity against the Barabási–Albert model [11, 12] that introduces the *'preferential attachment'* concept in the interpretation of the dynamics of large and complex networks. We formulate the model by setting up the following differential equation:

$$\frac{\partial k_j}{\partial t} = am\frac{1}{N-1} + (1-a)m\frac{k_i}{\sum_j k_j} \tag{7}$$

where k_j, k_i is the degree of the respective node (the number of edges starting from node j or i), t is the time step, m is the number of links that the Nth node will establish, $N-1$ the number of existing nodes of the cluster, and a is a parameter that takes values such that $0 < a < 1$; when $a = 0$ ($a = 1$) it is the *companionship (individuality)* that controls the direction of the added links. In other words the left term from the right-hand part of equation (7) accounts for the expression of *individuality* as 'links will be placed randomly', whereas the right term accounts for the expression of *companionship* as 'links will be directed to the node with the highest degree'. The above equation (7) can be produced by the following queuing process; when a new node is added to the network, the m edges that it carries are placed one by one as follows: the first node is placed randomly with probability p_1 or following the *'preferential attachment'* mechanism with probability $1-p_1$. Next, the second edge is placed following the same scheme but with probability p_2, and so on. It can be shown that $a = (1/m)\sum_{i=1}^{m} p_i$ and equation (7) holds. The solution of this differential equation leads to the following:

$$k_j(t) = \frac{1-\beta}{\beta}m\left(\frac{t}{t_i}\right)^{\beta} - \frac{1-2\beta}{\beta}m \tag{8}$$

where $\beta = (1-a)/2$ and t_i is the moment of insertion of the new node. Hence,

$$\frac{1-\beta}{\beta}m\left(\frac{t}{t_i}\right)^{\beta} \gg \frac{1-2\beta}{\beta}m. \tag{9}$$

Then equation (8) is reduced to

$$k_j(t) = \frac{1-\beta}{\beta}m\left(\frac{t}{t_i}\right)^{\beta}. \tag{10}$$

That is, the distribution of degrees is given as

$$P(t) \backsim 2\left(\frac{1-\beta}{\beta}\right)^{1/\beta} k^{(1+1/\beta)} \tag{11}$$

and, thus, the network is evolved by following a power law defined as

$$\gamma = 1 + \frac{1}{\beta} \qquad \text{or} \qquad \gamma = 1 + \frac{2}{1-a} > 3, \qquad \text{for } a \neq 1. \tag{12}$$

In the extreme when $a = 1$ the model reduces to a random network [13], while for $a = 0$ the model coincides with the Barabási–Albert model [11]. The above described formalism enables us to produce *undirected scale free networks* with $3 < \gamma < \infty$. An illustrative example comes from the *preferential attachment in sexual networks*. In this particular case the value of $\gamma = 3.4$ is the largest observed [12, 14, 15] resulting in $a = 16.7\%$ or an expression of *companionship*, which in this case is translated as: 'the target sexual partner

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is the one with the highest number of sexual links'. This argument is further supported by the fact that although preferential attachment in sexual networks has been experimentally established [16], the individual heterogeneity in the inclination to find new partners was found essential for modeling experimental data [15].

5. Conclusions

In summary, by augmenting the Vázquez *et al* [9] model with two opposing *driving forces* we were able to simulate the abrupt nature of synchronization. In addition we have shown that since in the first moments after the performance most spectators applause at their own pace ($I_{\rm prio}$ is large), there is no synchronization. Later, when the value of $I_{\rm prio}$ has been decreased, spectators may act to express their *companionship*. Random clusters of partial synchronized spectators cause an abrupt increment of $C_{\rm prio}$ for the neighboring spectators, which in turn leads to 'global' synchronization. However, after some 'synchronized' period $I_{\rm prio}$ could be enhanced again, resulting in loss of synchronization. Last but not least the expression of *individualism* does not characterize some communities of the Eastern European countries, where *companionship* is the main driving force—thus ease of achieving synchronization; whereas the opposite is observed in many Western country communities [4].

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