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NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A

The unfolding procedure of the alpha particle spectra of the $^{58}Ni(n, \alpha)^{55}Fe$ reaction

C. Tsabaris [a], D.Vlachos [a], C.A. Kalfas[b,*]

[a] *National Technical University of Athens, 15773 Athens, Greece*
[b] *NCSR Demokritos, Institute of Nuclear Physics, 15310 Agia Paraskevi, Athens, Greece*

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Abstract

The correction of the alpha particle energy spectra which is required because of the alpha particle energy loss, due to the thickness of the sample, is described by an analytical-empirical retrospective relation. This relation can yield a spectrum as it was produced in the sample from a corresponding measured spectrum. The alpha decays of the reaction $^{58}Ni(n, \alpha)^{55}Fe$ were used as the source of the alpha particle spectra. The projectile neutrons were produced via the T(p,n) and D(d,n) reactions in the energy range from 2 to 9 MeV. The validity of our assumption for the correctness of the alpha particle spectra was checked by comparing the measured spectrum with the spectrum produced by folding the corrected spectrum with the response function of the system. The calculated folded spectrum appeared to be consistent with the experimental data of the measured spectrum. Typical results are given at the neutron energy of 8 MeV, where the measured data were accumulated from a telescope placed at 79° relative to the neutron beam direction.

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1. Introduction

The alpha particles produced in a sample by a nuclear reaction lose a part of their energy before escaping from the sample. This effect distorts the spectrum in the sample. If $S(E_0)$ is the number of alpha particles emitted in the sample at energy E_0 per unit energy interval and $M(E)$ is the number of alpha particles detected at energy E per unit energy interval, then the equation which correlates the $M(E)$ and $S(E_0)$ is given by the following expression [1–3]

$$M(E) = \int_0^\infty R(E, E_0) S(E_0)\, dE_0, \qquad (1)$$

where $R(E, E_0)$ is the response function of the system related to the foil thickness and gives the probability that an emitted alpha particle with energy E_0 will be measured in the energy range from E to $E + dE$. The response function of the detection system depends on the energy of the alpha particles, the depth in the sample where an alpha particle is produced and the energy resolution of the system. In the present case the resolution of the surface barrier detector was not taken into account since it was small (0.8%).

* Corresponding author. Tel.: +30 1 651 8770; fax: +30 1 651 1215; e-mail: kalfas@cyclades.nrcps.ariadne-t.gr.

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If the response function $R(E, E_0)$ is known, one can either obtain $M(E)$ from $S(E_0)$ by performing the integration of the Eq. (1) (folding) or, in an inverse procedure, try to determine $S(E_0)$ from $M(E)$ after solving the corresponding integral equation as explained in the next paragraph (unfolding). The unfolding procedure can be carried out numerically by using an appropriate code.

2. Description of the correction

The aim of the proposed correction method is to transform the experimental data of the Measured Spectrum (MS) to calculated data of the spectrum produced in the sample, called Calculated Source Spectrum (CSS). This transformation requires the knowledge of the response function of the system, which mathematically can be achieved by introducing the delta function as an $S(E_0)$ function (see Eq. (1)). Since a delta function represents a monoenergetic spectrum at energy E_0, called here Assumed Source Spectrum (ASS), one can calculate the Assumed Measured Spectrum (AMS) by summing the various delta functions that describe monoenergetic measured alpha particle spectra coming from different sample depths. Given the AMS, the $R(E, E_0)$ is determined according to the Eq. (1).

As stated before, the solution of the integral Eq. (1) in order to obtain $S(E_0)$ from $M(E)$ (i.e. unfolding) can be carried out numerically. This solution, however, will be provided by an approximation method and the validity of its results must be checked. This validity check was performed by comparing the experimental data of the MS with the Calculated Measured Spectrum (CMS), which is derived by folding the CSS with the $R(E, E_0)$ function.

In summary, first the response function of the system has to be determined, then the unfolding of the MS has to be performed in order to calculate the CSS, and finally the comparison of the MS with the CMS will test the degree of validity of the unfolding method. These steps are shown schematically at the Fig. 1.

2.1. Determination of the response function

The response function of the system is determined by studying the probability that an alpha particle

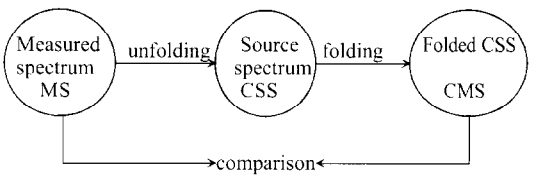

Fig. 1. Schematic drawing of the unfolding and folding procedures.

produced in the foil with energy E_0 will be detected with energy E. This probability is represented by the AMS, which can be calculated by using the stopping power ($\mathrm{d}E/\mathrm{d}x$) of the alpha particles in a sample. The stopping power is given numerically from the Beth-Bloch calculations derived from the code TRIM [4]. These values were fitted with the approximate Beth-Bloch formulae [5] in the alpha particle energy range from 2 to 10 MeV, as it is given in Eq. (2):

$$\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{b}{E + c}. \tag{2}$$

The function which relates the detected energy (E) with the produced one (E_0) is the following solution of the differential Eq. (2), under the condition: for $x = 0 \Rightarrow E = E_0$

$$E = -c + \sqrt{(E_0 + c)^2 - 2bx}. \tag{3}$$

As mentioned above, this probability is calculated by using a monoenergetic spectrum (ASS) as a source spectrum represented by the δ-function, i.e.

$$S(E_0) = \delta(E - E_0). \tag{4}$$

If the ASS is produced in a slice of thickness w, in depth x, then the respective AMS is yielded by the formulae (5):

$$\mathrm{d}M(E) = \delta(E - \sqrt{(E_0 + c)^2 - 2bx} + c)\frac{\mathrm{d}x}{w} \tag{5}$$

where $\mathrm{d}x/w$ is the probability that an alpha particle is produced in a depth between x and $x + \mathrm{d}x$. Integrating Eq. (5) over the thickness of the sample (w), the AMS is derived as follows:

$$M(E) = \frac{1}{w}\int_0^w \delta(E - \sqrt{(E_0 + c)^2 - 2bx} + c)\,\mathrm{d}x. \tag{6}$$

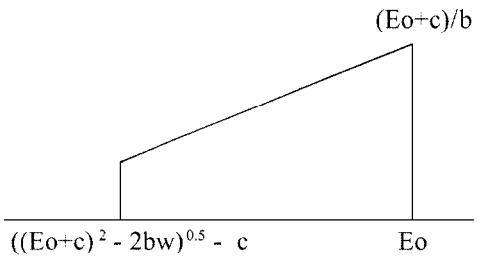

Fig. 2. The response function of the system in relative units.

The above integral will be solved by applying the following property of the delta function according to Eq. (7):

$$\delta(\varphi(x)) = \sum \frac{\delta(x - x_i)}{|\varphi'(x_i)|} \tag{7}$$

where x_i are the roots of the function $\varphi(x)$. The root of the function in the integral of Eq. (6) is

$$x_0 = \frac{(E_0 + c)^2 - (E + c)^2}{2b} \tag{8}$$

and finally the AMS, which for monoenergetic particles is equivalent to the response function of the system, is given from Eq. (9) and is shown in the Fig. 2.

$$M(E) = \begin{cases} \dfrac{E + c}{wb} & \text{if } 0 < x_0 < w, \\ 0 & \text{otherwise.} \end{cases} \tag{9}$$

The introduction of $u_{E_0}(E)$ function

$$u_{E_0}(E) = \begin{cases} 1 & \sqrt{(E_0 + c)^2 - 2bx} - c < E < E_0, \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

simplifies the mathematical description of the AMS, which is now expressed by the formulae

$$M_{E_0}(E) = u_{E_0}(E)\frac{E + c}{wb} \tag{11}$$

for a sole alpha particle.

2.2. Folding and unfolding

In the case that the source spectrum is not monoenergetic, the MS is derived by using Eqs. (1) and (11) and is expressed by formulae (12):

$$M(E) = \int_0^\infty M_{E_0}(E)\,\mathrm{d}E_0$$

$$\Rightarrow M(E) = \int_0^\infty S(E_0)u_{E_0}(E)\frac{E + c}{wb}\,\mathrm{d}E_0. \tag{12}$$

Clearly, the above integration (folding) can be easily performed and the results of the folding procedure are reliable.

The problem now is to obtain the CSS ($S(E_0)$) knowing the MS ($M(E)$) and the response function, i.e. to solve the above integral Eq. (12). The method that we followed for this integral equation is described in the Appendix A. This solution is provided by the following equations:

$$S\left(\sqrt{(E + c)^2 + 2bw} - c\right) = \left[\frac{\mathrm{d}G(E)}{\mathrm{d}E} + S(E)\right]\frac{\sqrt{(E + c)^2 + 2bw}}{E + c} \tag{13}$$

where

$$\frac{\mathrm{d}G(E)}{\mathrm{d}E} = \frac{-bw}{(E + c)^2}M(E) + \frac{bw}{(E + c)}\frac{\mathrm{d}M(E)}{\mathrm{d}E}. \tag{14}$$

3. Unfolding of the alpha decay measurements of the ^{58}Ni(n, α)^{55}Fe reaction

Eq. (13) gives the CSS as it was produced in the foil. A code named UNFOLDING has been written in C-language and is based on the algorithm described by Eq. (13).

The code uses double differentiation and certain criteria to identify the hidden peaks. One of the criteria, e.g. is the minimum energy distance between each identified peak and the experimentally determined cut-off energy. This method, although very powerful in peak identification, has a certain limitation: peaks at the low energy part of the detected spectrum are superimposed on the tail of the higher energy peaks. This causes a larger error in the identification process due to the additional uncertainty introduced in the double differentiation. It should also be noted that very small peaks identified by this method are of limited reliability.

The width of the identified peaks thus produced depends mostly on the uncertainty of the method

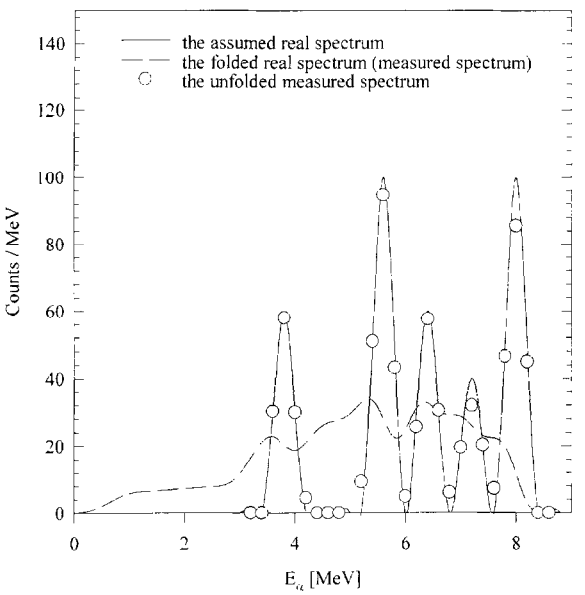

Fig. 3. Demonstration of the peak identification reliability of the method.

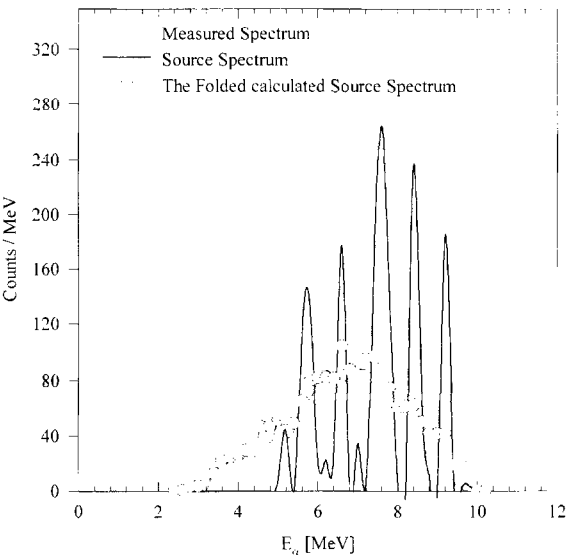

Fig. 4. The measured spectrum $M(E)$ (thin line), the calculated source spectrum $S(E)$ (thick line) and the folded calculated source spectrum $M'(E)$ (circles).

and is usually much larger than the FWHM of the silicon detector used (see also limitation in the next Section 4). To demonstrate the reliability of this method, an artificial set of random alpha peaks was constructed ("assumed real spectrum") and the subsequent "folded spectrum" was subject to this identification method as shown in Fig. 3. The reproduction of the original random peaks is clearly seen.

Some examples of running this code will be given below for the correction of the alpha particle measurements [5, 6] of the ^{58}Ni(n, α)^{55}Fe reaction. The thickness of the ^{58}Ni foil was 4.0 mg/cm^2 and the fit parameters for the calculation of the energy loss in the Ni sample (see Eq. (2)) are: $b = 4.60$ (MeV cm)^2/mg, $c = 4.72$ MeV.

Some results of running the programme UNFOLDING are shown in the Fig. 4. First, the measured alpha particle spectrum ($M(E)$) produced from the reaction ^{58}Ni(n, α)^{55}Fe at neutron energy 8 MeV is plotted (see thin line in Fig. 4). This spectrum was accumulated for 16 h from a telescope placed at 79° relative to the neutron beam direction. Subsequently, the CSS ($S(E_0)$) is shown as well (thick line). The total number of the alpha particles (number of counts) of the spectra $M(E)$ and $S(E_0)$ is equal, as expected. This was a condition of this calculation and it can be easily verified, if one integrates Eq. (12) over all alpha particle energies from 0 to infinity (see Appendix B).

The validity of the unfolding procedure was checked by using the inverse operation (folding). The folding of the CSS ($S(E_0)$) with the response function gives the spectrum as it was measured (CMS). The comparison of the MS and CMS shows a substantial degree of similarity (see Fig. 4). This similarity supports the reliability of our proposed method in correcting measured alpha particle spectra with the above unfolding procedure.

The uncertainty of the detected alpha particle energy, due to the energy resolution of the detectors, does not cause any errors in the calculation of the energy loss of the alpha particles, because of the following reasons:

a. the stopping power (dE/dx) is well defined by fitting (see Eq. (2)) the calculated values derived from the Beth–Bloch formulae in the alpha particle energy range from 2 to 10 MeV [5].
b. the variation of the stopping power (dE/dx) as a function of the alpha particle energy is very smooth, since the energy range of the emitted alpha particles varies from 6 to 10 MeV [5, 6].

4. Discussion

In the past, it was impossible to present double differential cross section data since the used foils were thick enough for alpha particles distorting the spectrum as it was produced in the nucleus of the foil. However, with the proposed correction procedure this problem can be overcome. The double differential cross section data of the ^{58}Ni(n, α)^{55}Fe reaction in the neutron energy range from 2 to 9 MeV were corrected and presented in a previous work [5].

The unfolding of the alpha particle spectra is performed by taking into account the following assumptions:

- The detector is a point detector.
- The alpha particles are produced in the central axis of the sample.
- The response function of the detector is not taken into account because the resolution of the surface barrier was small (0.8%).
- The alpha particles have always the same direction (the line between sample and detector) in each collision with the nuclei of the Nickel sample.

In conclusion, the proposed correction method offers a rather reliable procedure which

a. can be used to obtain the energies of the alpha particles emitted in a reaction experiment and
b. will make it possible to obtain double differential cross section information in such experiments involving alpha particle emission.

It should be noted that the agreement between the MS and CMS in the high energy part starts failing and this is an inherent problem of the method. It should be worthwhile to achieve a more reliable correction making a Monte Carlo simulation taking into account all the assumptions mentioned above.

Appendix A

The integral Eq. (12) is solved by using a transformation as well as the identities of the delta function. The transformation formulae is given according to Eq. (A.1) and the identities of the delta function were given before from Eq. (7).

$$G(E)=\frac{wb}{E+c}M(E) \tag{A.1}$$

substituting the *measured* spectrum from Eq. (12) the following equation is deduced

$$G(E)=\int_0^{\infty} S(E_0)u_{E_0}(E)\,\mathrm{d}E_0. \tag{A.2}$$

Differentiating Eq. (A.2) over measured energy (E) and using also the identity of the theta function ($\mathrm{d}\theta(x)/\mathrm{d}x=\delta(x)$) we deduce the following equations:

$$\frac{\mathrm{d}G(E)}{\mathrm{d}E}=\int_0^{\infty} S(E_0)\frac{\mathrm{d}}{\mathrm{d}E}[\theta(E-\sqrt{(E_0+c)^2-2bx}+c)-\theta(E-E_0)]\mathrm{d}E_0, \tag{A.3}$$

$$\frac{\mathrm{d}G(E)}{\mathrm{d}E}=\int_0^{\infty} S(E_0)[\delta(E-\sqrt{(E_0+c)^2-2bx}+c)-\delta(E-E_0)]\mathrm{d}E_0. \tag{A.4}$$

Using now the identity of the delta function as given from Eq. (7), Eq. (A.5) is deduced.

$$\frac{\mathrm{d}G(E)}{\mathrm{d}E}=\int_0^{\infty} S(E_0)\frac{\delta(E_0-\sqrt{(E+c)^2+2bx}+c)}{\frac{\sqrt{(E+c)^2+2bw}}{E+c}}\times\mathrm{d}E_0-S(E), \tag{A.5}$$

$$\frac{\mathrm{d}G(E)}{\mathrm{d}E}=\frac{E+c}{\sqrt{(E+c)^2+2bw}}\times S(\sqrt{(E+c)^2+2bw}-c)-S(E), \tag{A.6}$$

$$S(\sqrt{(E+c)^2+2bw}-c)=\left[\frac{\mathrm{d}G(E)}{\mathrm{d}E}+S(E)\right]\frac{\sqrt{(E+c)^2+2bw}}{E+c}, \tag{A.7}$$

where

$$\frac{\mathrm{d}G(E)}{\mathrm{d}E}=\frac{-wb}{(E+c)^2}M(E)+\frac{wb}{(E+c)}\frac{\mathrm{d}M(E)}{\mathrm{d}E}. \tag{A.8}$$

Appendix B

$$\int_0^\infty M(E)\,\mathrm{d}E$$
$$= \int_0^\infty \frac{E+c}{wb}\left(\int_0^\infty S(E_0)u_{E_0}(E)\,\mathrm{d}E_0\right)\mathrm{d}E$$
$$= \int_0^\infty S(E_0)\frac{1}{w}\left(\int_0^\infty \frac{E+c}{b}u_{E_0}(E)\,\mathrm{d}E\right)\mathrm{d}E_0$$
$$= \frac{1}{w}w\int_0^\infty S(E_0)\,\mathrm{d}E_0$$
$$\Rightarrow \int_0^\infty M(E)\,\mathrm{d}E = \int_0^\infty S(E_0)\,\mathrm{d}E_0.$$

We used the result of the integral

$$\int_0^\infty \frac{E+c}{b}u_{E_0}(E)\,\mathrm{d}E$$
$$= \int_{\sqrt{(E_0+c)^2-2bw}-c}^{E_0} \frac{E+c}{b}\,\mathrm{d}E = w.$$

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