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The use of vertical and horizontal accelerations of a floating buoy for the determination of directional wave spectra in coastal zones

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Abstract

A parametric method is presented for calculating the directional wave spectrum from vertical and horizontal accelerations of a floating buoy. These measurements are obtained from accelerometers attached on board the buoys of the POSEIDON network. The method assumes the superposition of two independent wave trains giving a better approach to the multidirectional nature of the wave field. A general purpose software module called WaveAna has been developed which can process horizontal and vertical displacements of floating buoys. The functionality of WaveAna is shown with the measured data from the POSEIDON network and the determination of wind generated waves and swell is demonstrated.

Keywords: Sea wave spectra

1. Introduction

Surface ocean waves are commonly described by their directional spectrum, which gives the distribution of wave amplitude with frequency and direction. The knowledge of the wave directional spectrum is fundamental for various marine applications (safety at sea, offshore industry, ship design, ship routine, coastal engineering, survey and protection of coastal environment). Furthermore, most of the wave prediction models, which provide forecast of sea state over ocean basins for several days are based on the concept of directional wave spectra. In situ observation systems are developed and used to provide directional wave spectra. These observations can be used to validate or constrain these models. They can also be used as local observations for the various marine applications. Information on surface waves can now also be obtained from remote sensing techniques (mainly radar). The main advantage of this technique is that it provides information on the wave field in regions and over scales, which cannot be covered by in situ instruments. Although remote sensing is very promising, the analysis techniques still need assessment, so that there is a need of coincident in situ measurements.

The most widely used and accepted method of studying wind generated waves is an examination of the spectra at a single point. This approach is based mostly on the assumption that recorded time series of the surface elevation, pressure or velocities are the linear superposition of small oscillations regardless of their directions of propagation [1].

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Due to complicated energy transfers from the atmosphere to the sea, the resulting surface waves are multidirectional, with only some of the waves aligned with the wind direction. Wave multidirectionality is also a result of the superposition at a given point of a number of wave trains which may be generated by different and remote atmospheric forcing systems.

Analysis of raw data is based on the so-called 'stochastic model' [2]. In this approach, the sea surface elevation is considered to be a Gaussian random variable (linear model). One of the most common techniques used for the calculation of directional wave spectra is the use of accelerometers or inclinometers on board the buoys which measure the heave acceleration or the vertical displacement of the buoy hull during the wave acquisition time. A Fast Fourier Transform (FFT) is applied to the data by the processor usually on board the buoy to transform the data from the temporal domain into the frequency domain. Note that the raw acceleration or displacement measurements may or may not transmitted shore-side. Post-processing techniques are then performed on the transformed data to account for both hull and electronic noise. It is from this transformation that non-directional spectral wave measurements (i.e., wave energies with their associated frequencies) are derived. Along with the spectral energies, measurements such as significant wave height (Hm0), average wave period (SprTp), and dominant period (Mdir) are also derived from the transformation.

In this paper, a parametric method is presented for calculating the directional wave spectrum from vertical and horizontal accelerations of a floating buoy. These measurements are obtained from accelerometers attached on board the buoys of the POSEIDON network [3]. The method assumes the superposition of two independent wave trains giving a better approach to the multidirectional nature of the wave field. A general purpose software module called WaveAna has been developed which can process horizontal and vertical displacements of floating buoys. The functionality of WaveAna is shown with the measured data from the POSEIDON network and the determination of wind generated waves and swell is demonstrated.

2. Formulation of the problem

The following analysis for the determination of the directional wave spectra can be applied when the wave sensors give the vertical and horizontal accelerations of the floating buoy. In these cases, the accelerations can be twice integrated and thus can be transformed in vertical and horizontal shifts. According to Linear Wave Theory [4], the directional wave spectra $E(\omega, \theta)$ is given by

$$E(\omega, \theta) = S(\omega) \cdot D(\omega, \theta), \tag{1}$$

where $S(\omega)$ is the total wave power at frequency ω and $D(\omega, \theta)$ is the distribution of this power over all possible directions. Notice here that the integral of $D(\omega, \theta)$ over all possible directions is normalized to unity. It is reasonable to assume here that the main direction of wave propagation is that of the generating wind (since we are talking for waves in coastal zones). But the experiment shows that this is not true. The wave power is distributed in all directions located near a dominating one. A parametric model for this distribution is given [5,6]

$$D(\omega, \theta) = \hat{D}(\omega) \cdot \cos^{2p(\omega)} \left(\frac{\theta - \theta_0}{2} \right), \tag{2}$$

where θ_0 is the main direction and $p(\omega)$ is an integer which depends on frequency. This model describes well the local wind generating waves but fails when the wave field contains propagating waves from other directions like swells.

In order to account for this problem, we assume that the wave field is is composed by a local wind generated one (located at direction θ_0) and a propagating field (located at direction θ_1). Thus, the directional spectrum can be expressed by [7]:

$$D(\omega, \theta) = \Delta_0(\omega) \cdot \cos^{2p(\omega)} \left(\frac{\theta - \theta_o}{2} \right) + \Delta_1(\omega) \cdot \cos^{2p(\omega)} \left(\frac{\theta - \theta_1}{2} \right). \tag{3}$$

The exponent $p(\omega)$ is considered to be an integer and the same for the two components of the wave field. This is necessary, since otherwise it will result in an indefinite system of equations.

Expression (3) can be used with different types of measurements, like accelerations, velocities, translations or angular velocities, in order to estimate the wave spectra. In our cases, the measurements that the POSEIDON network [3] produces are translations of the floating buoy in three directions, the normal (subscript n), the horizontal on the North–South axis (subscript x) and the horizontal on the East–West axis (subscript y). The auto- and cross-correlated spectra S_{ij} can be calculated:

$$C_{nn}(\omega) = \int_0^{2\pi} E(\omega,\theta)\mathrm{d}\theta = \int_0^{2\pi} D(\omega,\theta)S(\omega)\mathrm{d}\theta = S(\omega) \tag{4}$$

$$Q_{nx}(\omega) = \int_0^{2\pi} E(\omega,\theta)\frac{g}{\omega^2}k\cos\theta\mathrm{d}\theta = C_{nn}\frac{gk}{\omega^2}\int_0^{2\pi} D(\omega,\theta)\cos\theta\mathrm{d}\theta \tag{5}$$

$$Q_{ny}(\omega) = \int_0^{2\pi} E(\omega,\theta)\frac{g}{\omega^2}k\sin\theta\mathrm{d}\theta = C_{nn}\frac{gk}{\omega^2}\int_0^{2\pi} D(\omega,\theta)\sin\theta\mathrm{d}\theta \tag{6}$$

$$C_{xx}(\omega) = \int_0^{2\pi} E(\omega,\theta)\frac{g^2}{\omega^4}k^2\cos^2\theta\mathrm{d}\theta = C_{nn}\frac{g^2k^2}{\omega^4}\int_0^{2\pi} D(\omega,\theta)\cos^2\theta\mathrm{d}\theta \tag{7}$$

$$C_{yy}(\omega) = \int_0^{2\pi} E(\omega,\theta)\frac{g^2}{\omega^4}k^2\sin^2\theta\mathrm{d}\theta = C_{nn}\frac{g^2k^2}{\omega^4}\int_0^{2\pi} D(\omega,\theta)\sin^2\theta\mathrm{d}\theta \tag{8}$$

$$C_{xy}(\omega) = \int_0^{2\pi} E(\omega,\theta)\frac{g^2}{\omega^4}k^2\cos\theta\sin\theta\mathrm{d}\theta = C_{nn}\frac{g^2k^2}{\omega^4}\int_0^{2\pi} D(\omega,\theta)\cos\theta\sin\theta\mathrm{d}\theta, \tag{9}$$

where

$$S_{ij} = C_{ij} - iQ_{ij}. \tag{10}$$

Using expression (3) the above expression becomes:

$$C_{nn} = I(p)(\Delta_0 + \Delta_1) \tag{11}$$

$$C_{xx} = \frac{k^2g^2}{\omega^4}\cdot I(p)\cdot\left[\Delta_0\left(\frac{1}{2}+\frac{p(p-1)\cos 2\theta_0}{2(p+1)(p+2)}\right)+\Delta_1\left(\frac{1}{2}+\frac{p(p-1)\cos 2\theta_1}{2(p+1)(p+2)}\right)\right] \tag{12}$$

$$C_{yy} = \frac{k^2g^2}{\omega^4}\cdot I(p)\cdot\left[\Delta_0\left(\frac{1}{2}-\frac{p(p-1)\cos 2\theta_0}{2(p+1)(p+2)}\right)+\Delta_1\left(\frac{1}{2}-\frac{p(p-1)\cos 2\theta_1}{2(p+1)(p+2)}\right)\right] \tag{13}$$

$$C_{xy} = \frac{k^2g^2}{\omega^4}\cdot I(p)\cdot\left(\Delta_0\frac{p(p-1)\sin 2\theta_0}{2(p+1)(p+2)}+\Delta_1\frac{p(p-1)\sin 2\theta_1}{2(p+1)(p+2)}\right) \tag{14}$$

$$C_{nx} = \frac{kg}{\omega^2}\cdot I(p)\cdot\left(\Delta_0\frac{p\cdot\cos\theta_0}{p+1}+\Delta_1\frac{p\cdot\cos\theta_1}{p+1}\right) \tag{15}$$

$$C_{ny} = \frac{kg}{\omega^2}\cdot I(p)\cdot\left(\Delta_0\frac{p\cdot\sin\theta_0}{p+1}+\Delta_1\frac{p\cdot\sin\theta_1}{p+1}\right), \tag{16}$$

where n stands for the vertical and x, y for the horizontal components. The function $I(p)$ is given by

$$I(p) = 2\pi\frac{1\cdot 3\cdot 5\ldots(2p-1)}{2\cdot 4\cdot 6\ldots 2p}\cdot\frac{p}{p+1}. \tag{17}$$

3. The software module WaveAna

A software module called WaveAna has been developed for the analysis of measured data. The user interface is shown in Fig. 1. The algorithm used for the solution of Eqs. (11)–(16) is the following:

- The value of parameter p takes only integer values between 2 and 60 while the angles θ_0 and θ_1 take values between 0 and 355 degrees with 5° steps.
- The parameters Δ_0 and Δ_1 are calculated for every triplet (p, θ_0, θ_1) using Eqs. (4) and (5).
- the values $(p, \theta_0, \theta_1, \Delta_0, \Delta_1)$ are substituted in Eqs. (6)–(8) and the total error is calculated

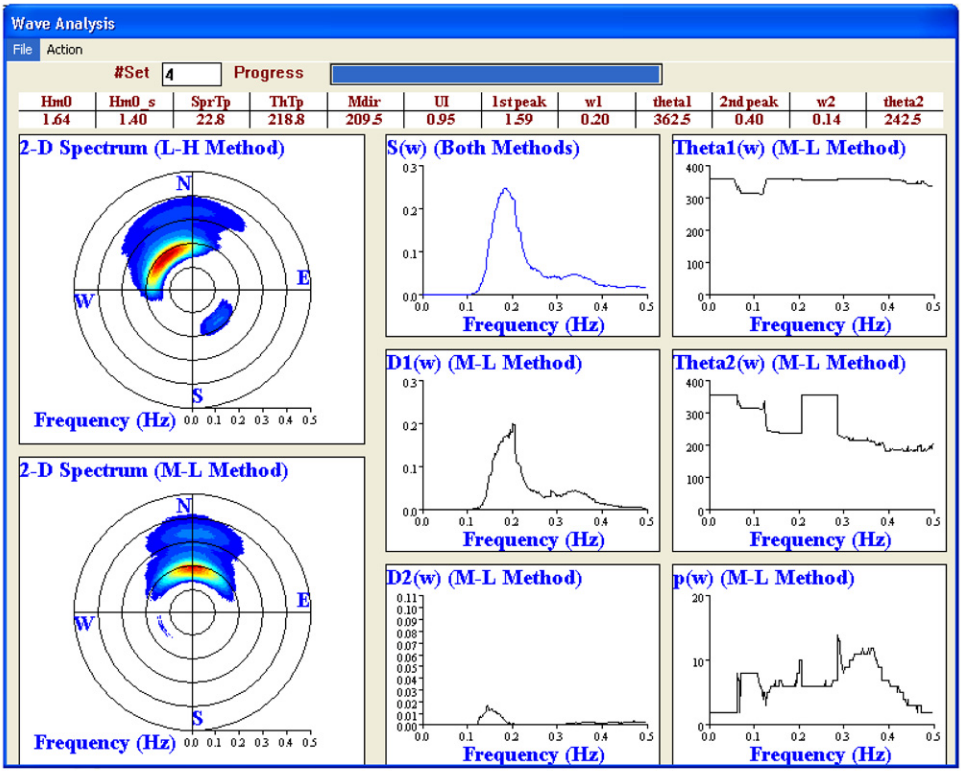

Fig. 1. User interface of the WaveAna software package for the calculation of directional wave spectra.

- From all the combinations, the solution that minimizes the total error is selected.
- In the case where the two angles are the same, the system is solved with the assumption that $\Delta_1 = 0$.
- When the sum of the angles is 360°, the solution is taken from Eqs. (4) and (6).
- Finally, a constraint for the solution is that the direction of the waves must be a smooth function of frequency.

The above procedure is continued for every frequency, so at the end of the algorithm the functions $\Delta_0(\omega)$ and $\Delta_1(\omega)$ have been calculated. The directional wave spectrum $D(\omega, \theta)$ is calculated from Eq. (3).

WaveAna reads the measured values from a text file which contains the vertical, North–South and East–West displacements of the buoy in three columns. The user, through the Setup procedure, gives the file name which contains the measurements, the sampling period in seconds, the number of samples (rows) for a full measurement interval, the averaging length for smoothing the measurements and the depth at the point of measurement.

All data have been obtained from the POSEIDON system [3]. The analysis showed that there are cases where local wind generated waves can be easily separated from propagating ones.

4. Numerical results

Two events will be presented which show the capability of WaveAna to analyze the measured displacement of a floating buoy. During the first event shown in Fig. 2, WaveAna plots the estimated directional spectrum every 3 h. The time at which the spectra are calculated increases from left to right and from top to bottom. Table 1 shows the calculated significant wave height, main frequency and main direction of the two waves as estimated from WaveAna. At the beginning there is a weak wind field which causes WaveAna to produce a wide angular spread of the spectrum. During the second and third spectrum, it seems that a new wave is generated at 230° while there is another one wave field at 360°. The high frequency of the second wave field (see Table 1) leads us to the conclusion that this wave field is more likely noise than wind generated wave. In the third spectrum it is clearly seen that the wind generated wave at 230° dominates. Later, during the fourth, fifth and sixth spectrum, the frequency of the first wave at 230°

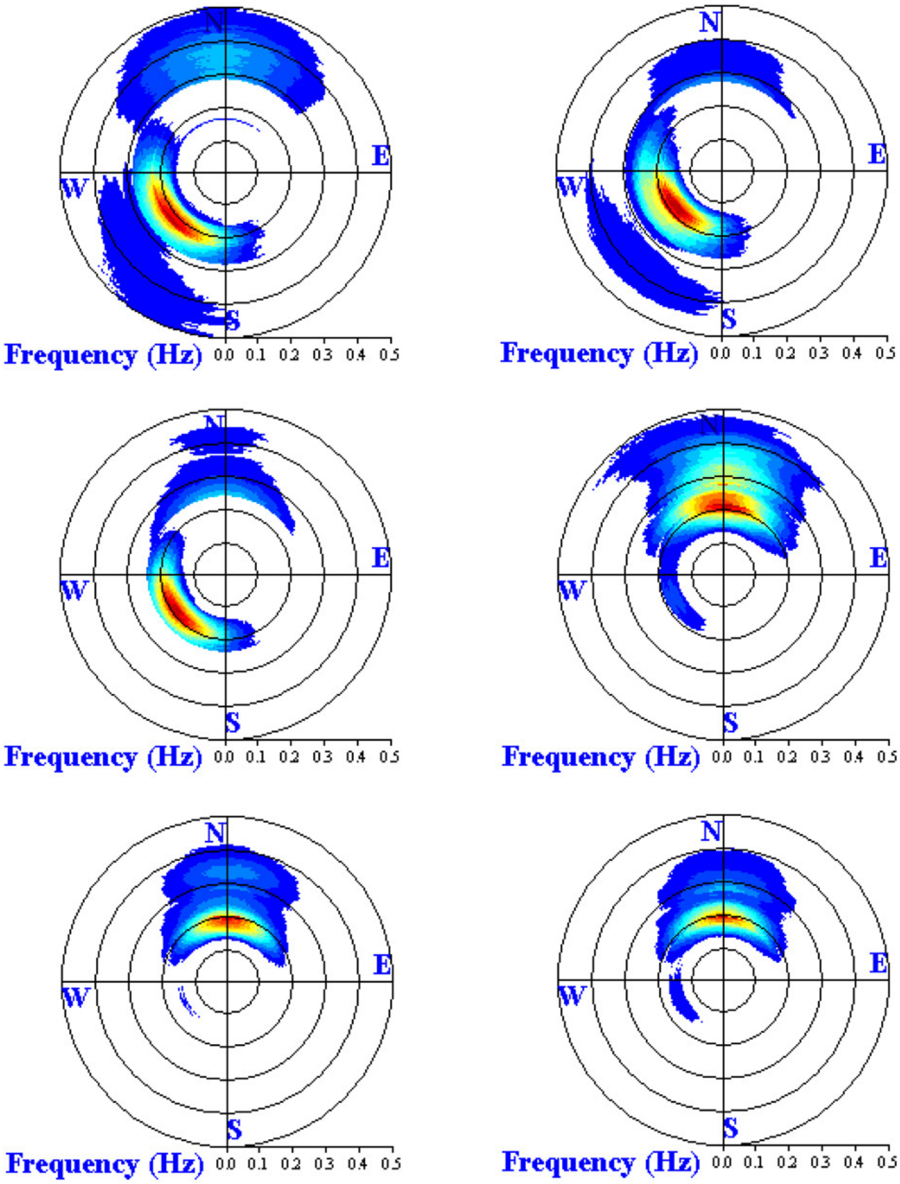

Fig. 2. Directional wave spectra calculated by the WaveAna software every three hours for the first event. The time at which the spectra are calculated increases from left to right and from top to bottom.

decreases. This behavior is related to a traveling wave rather than to a wind generated one. At the same time, a new wave is generated at 360° as can be seen both from the wave height and the frequency. Finally, the new wave at 360° dominates the spectrum. During this event, the evolution of the wind field as measured by the floating buoy is fully correlated with the generation of the waves, while the contribution of the traveling wave during the last three spectra is more than 20% of the total wave height. This contribution is important in wave height prediction efforts based on the existing wind.

The second event is shown in Fig. 3 and the estimated parameters in Table 2. In this event there is a strong turning wind from 190° to 360°. At the beginning there is a strong wind generated wave at 180°. The frequency of this wave starts to decrease which means that this wave becomes a traveling one, while a second wave is generated between 300° and 360°. Finally, the wave at 360° dominates the spectrum. The contribution of the traveling wave in this event can be more than 40% of the total wave, which can lead to serious underestimation of the wave height.

Table 1
Measured wave parameters corresponding to the event plotted at Fig. 2

Time	$Hm0_1$	ω_1	θ_1	$Hm0_2$	ω_2	θ_2
0	0.65	0.22	225	0.42	0.29	360
3	0.74	0.19	225	0.32	0.27	360
6	0.82	0.2	230	0.47	0.24	360
9	0.4	0.17	235	1.24	0.22	360
12	0.4	0.14	240	1.6	0.2	360
15	0.46	0.15	245	1.57	0.2	360

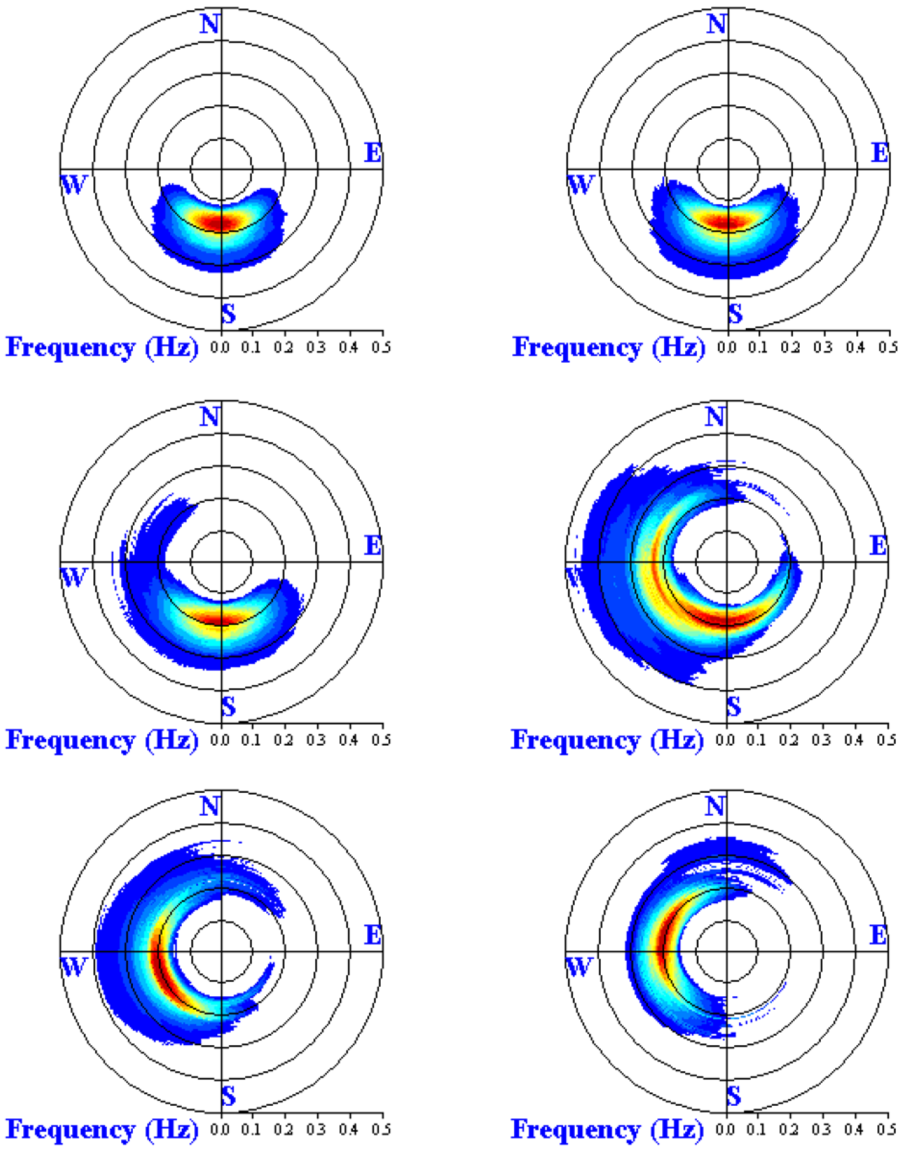

Fig. 3. Directional wave spectra calculated by the WaveAna software every three hours for the second event. The time at which the spectra are calculated increases from left to right and from top to bottom.

Finally, the calculation of the wind generated wave can be very useful in determining the exact response of the sea state in a certain wind. The left part of Fig. 4 shows the location of a measuring station while the right part of the same

Table 2
Measured wave parameters corresponding to the event plotted at Fig. 3

Time	$Hm0_1$	ω_1	θ_1	$Hm0_2$	ω_2	θ_2
0	3.15	0.19	180	0.57	0.39	180
3	2.53	0.19	180	0.8	0.22	300
6	1.93	0.19	180	0.9	0.2	300
9	1.26	0.18	180	1.3	0.23	275
12	1.0	0.18	190	1.8	0.21	260
15	0.65	0.17	200	1.7	0.21	380

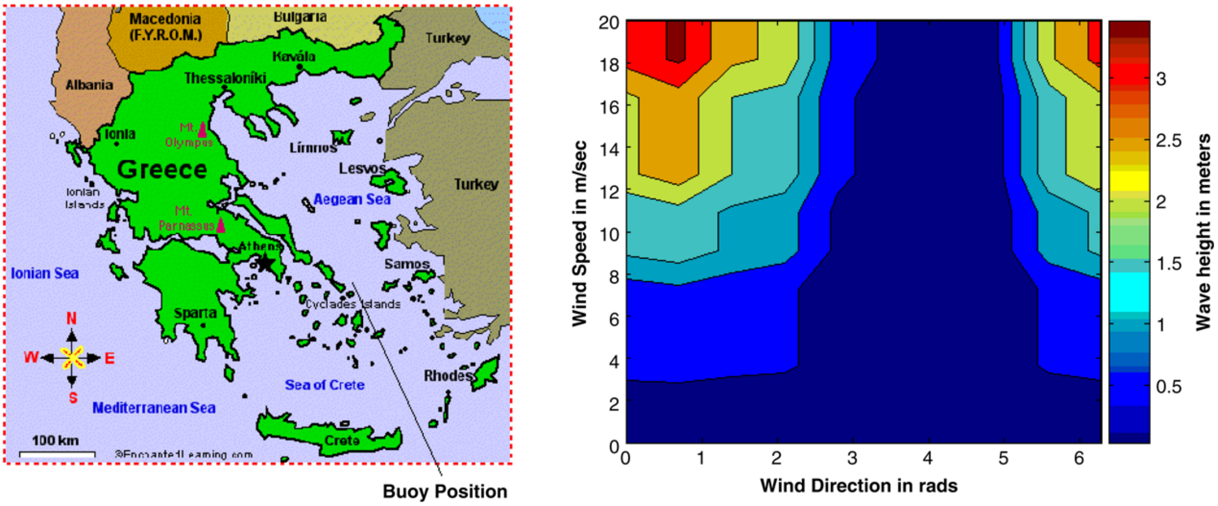

Fig. 4. Wave map produced by the calculated wind generated wave for a specific location. Data have been collected for a period of 5 years.

figure shows the response of the wind generated wave in this location to wind at different velocities and directions. The data have been collected for a period of five years with wave height and wind measurement every three hours. The x-axis corresponds to the wind direction measured in rads and the y-axis to the wind speed. The coloring of an area at position ($theta$, v) corresponds to the wind generated wave height produced in this location when the wind is coming from direction *theta* with speed v. Such maps can be very useful in navigation and in the procedure of data assimilation in wave models.

5. Conclusions

The estimation of the directional wave spectrum is very important in cases where both wind generated and traveling waves contribute to the total wave height. Wave height prediction based on existing wind can produce significant errors if existing traveling waves are neglected. A practical method to distinguish between traveling and wind generated waves is to assume that the total wave is the contribution of two independent waves at different frequencies and at different directions. WaveAna is a user friendly software which using this assumption can calculate the directional wave spectrum and the main parameters which characterize the two waves. Numerical tests performed in measured data obtained from the POSEIDON network, show that the estimations produce by WaveAna are fully correlated with the existing wind and that the contribution of the traveling waves can be calculated. Moreover, the calculated wind generated wave can be used to produce wave maps which locally characterize areas of interest (like ship passings) by drawing the response of the waves to wind at different velocities and directions.

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References

[1] S.R. Massel, Ocean Surface Waves: Their Physics and Prediction, World Scientific, Singapore, 1996.
[2] J. Allender, T. Audunson, S.F. Barstow, S. Bierken, H.E. Krogstad, P. Steinbakke, L. Vartdal, L.E. Borgman, C. Graham, Ocean Engineering 16 (5–6) (1989) 505.
[3] T. Soukissian, G.Th. Chronis, K. Nittis, Sea Technology 40 (7) (1999) 31.
[4] M.S. Longuet-Higgins, D.E. Cartwright, N.D. Smith, Proc. Ocean Wave Spectra 111 (1963).
[5] M.L. Heron, J. Phys. Oceanography 17 (1987) 281.
[6] J.A. Ewing, A.K. Laing, J. Phys. Oceanography 17 (1987) 1696.
[7] D.E. Hasselman, M. Dunckel, J.E. Ewing, J. Phys. Oceanography 10 (1980) 1264.